Slope stability in relation to the coefficient of thermal conductivity

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Abstract: Rock slopes gradually become unstable with time and may finally collapse. The objective of this study is to investigate the deterioration of a rock slope at atmospheric temperature. Rock may deteriorate due to many causes, such as, atmospheric temperature, rainfall, and so on. This study is focused on the effect of atmospheric temperature on rock slope stability. Slope stability was quantitatively evaluated in relation to the change of the atmospheric temperature. A linear heat conduction equation including temperature distribution as a functional formula equation was used in this study. The deteriorated depth in the slope was estimated, and numerical analysis was conducted to calculate the safety factor of the slope. Five slopes were selected for this study, the temperature distribution during a two-year period was used for a trigonometric function. The unique constant was derived on the basis of the regional characteristic of the temperature distribution. The types of rock in this study were sandstone, granite and gneiss. Three physical parameters, the coefficient of thermal conductivity, specific heat and dry density were used to predict the deteriorated depth of three rock masses using the linear heat conduction analysis. The change of temperature in the rock mass was investigated at four time stages. From the results of analysis, the depth deteriorated by atmospheric temperature was predicted by region and rock type. It has typically been known that the constant temperature zone in the rock mass exist over 50m from the rock surface. Therefore, the temperature change in the rock mass was analyzed up to 50m from rock surface to predict the deteriorated depth of rock mass. It is possible that the change of slope stability by deterioration with time can be assessed quantitatively in this way.

Résumé: Le puits incliné au rocher devient graduellement instable avec le temps et pourrait enfin s'effondrer. Cette étude a pour objectif d'investiguer la détérioration d'un puits incliné au rocher à la température atmosphérique. Le rocher pourrait se détériorer pour plusieurs raisons telles que température atmosphérique, précipitation, etc. Cette étude se focalise sur la conséquence de la température atmosphérique sur la stabilité de puits incliné au rocher. La stabilité du puits a été quantitativement évaluée en fonction du changement de la température atmosphérique. L'équation de la conduction thermique linéaire comprenant la distribution de température comme une équation de formule fonctionnelle a été utilisée dans cette étude. La profondeur détériorée dans le puits a été estimée, et l'analyse numérique s'est déroulée pour calculer la facteur de sécurité du puits. Cinqs puits ont été sélectionnés pour cette étude, la distribution de température pendant deux ans a été utilisée pour la fonction trigonométrique. La constante unique a été dérivée sur la base des caractéristiques régionales de la distribution de température. Les types de rocher dans cette étude étaient les grès, granite et gneiss. Les trois paramètres physiques, le coefficient de la conduction thermique, la chaleur spécifique et la masse volumique sèche ont été utilisés pour prévoir la profondeur détériorée de trois rochers à travers l'analyse de la conduction thermique linéaire. Le changement de température dans le rocher a été investigué en quatre temps. A partir des résultats d'analyse, la profondeur détériorée par la température atmosphérique a été prévue par le type de région et rocher. Il est typiquement connu que la zone de température constante dans le rocher existe à 50m de la surface de rocher. Par conséquent, le changement de température dans le rocher a été analysé jusqu'à 50m de la surface de rocher afin de prévoir la profondeur détériorée de rocher. Il est possible que le changement de la stabilité de puits par la détérioration avec le temps soit évalué quantitativement dans ce sens..

Keywords: expansion, frozen ground, landslide, slope stability, thermal properties, weathering,

INTRODUCTION

Slope structures consist mainly of soil and rock. Constructed slopes, particularly when made in rock, are very stable during construction, but diminish in strength and deteriorate over time. Since the types of superficial slope collapses in Korea have been mostly analyzed, this present study is concerned with the estimation of the depth of deterioration from the exposed portion of a rock slope, depending on time. A quantitative analysis of rock deterioration is important in slope stability analysis, as different mechanical or physical properties need to be used in the analysis for fresh rocks and for different depths of deterioration. Numerous studies have been published on the effect of freeze thawing on soil-based foundation. Yoon et al. (2003) investigated the dynamic behaviour of weathered granite soil before and after freeze thawing; Graham et al. (2001) studied the effects of freeze thawing on the water conductivity of plastic clay. Li et al. (2002) studied the porosity and permeability of soil that is undergoing changes due to the effects of freeze-thawing. The influenced depth and the stability of rock slope due to freeze thawing were analyzed by Baek (2001). Among rock freeze thawing experiments, the one on tuff (Tamer et al., 2003) was intended to confirm conversely the strain inside the rock when ice crystals form inside the rock.

In the present study the depth that is influenced by freeze thawing was estimated using the first order thermal conductivity equation on rock. In order to reveal the deterioration characteristics of rock in Korea, we divided the country into five regions and selected five representative cities in the regions, and analyzed the temperature distributions for the past two years to yield characteristic functions, which were analyzed by region. In addition, we chose sandstone, granite, and gneiss, which occur substantially in Korea, to analyze the deterioration characteristics of each rock type.

METHODS

Temperature distributions in the five areas in Korea

The present study is intended to estimate the depth of rock exposed to freeze thaw that is influenced by external temperature and rock type, assuming temperature changes as a possible cause of deterioration. Since temperature changes differ in different areas, we first divided South Korea into five regions as representative temperature variation regions and selected five cities, i.e., Gangneung, Seoul, Daejeon, Pohang, and Gwangju, as indicated in Figure 1 The reason for the selection is that they geographically represent the regions despite the possibility of local temperature changes. In addition, latitude and inland/coast were considered in the selection to determine the depth of rock that is influenced by freeze thawing and temperature.

Figure 1. Study areas selected to analyze the influence of freezing and thawing on rock

In order to examine temperature changes in the five areas, we analyzed the daily mean temperatures for the past two years from meteorological data on the Korean Meteorological Administration web site. We extrapolated a curve from the analyzed daily mean temperature that can be expressed as the function (1), where the first term in the right-hand side denotes the mean of daily lowest temperatures, the second term denotes yearly largest change, and the third term denotes temperature variation in a day. Regional characteristic constants C_0 and C_1 that are determined by temperature change equation like (1) for each area are listed in Table 1.

$$\psi(t) = C_0 + C_1 \sin(2\pi \times \frac{t}{365}) + 0.1\sin(2\pi \times 24t) \tag{1}$$

Table 1. Constants of temperature distribution curves at the 5 study cities

Constant / Area	C0	C1
Gangneung	12.8	13
Seoul	13.2	15
Dajeon	13.6	14
Pohang	14.2	14
Gwangju	13.8	14

These constants, which are unique for each area, are determined by the temperature distribution in a particular area, and are intended to estimate the depth of rock in the area that is subject to the influence of deterioration depending on time. Figure 2 shows the temperature distribution and the extrapolated curve in each area for the past two years.

According to Baek (2005), areas that exhibit a large difference between the highest temperature and the lowest temperature can be assumed to have relatively more intense rock deterioration, and the temperature factor must be considered with regard to the slope stability of a tunnel mouth in the area. Hence for the five areas, the highest temperature, the lowest temperature, and the temperature difference between the two in the monthly mean temperature for the past two years are listed in Table 2. Among the five areas Seoul displays the largest temperature variation the highest temperature in August 2004 was 26.1 ºC and the lowest temperature in January 2003 was -2.5 ºC, and the temperature difference between the two was 28.6 ºC. Temperature variations in the other areas are in the downward order of Pohang, Daejeon, Gwangju, and Gangneung.

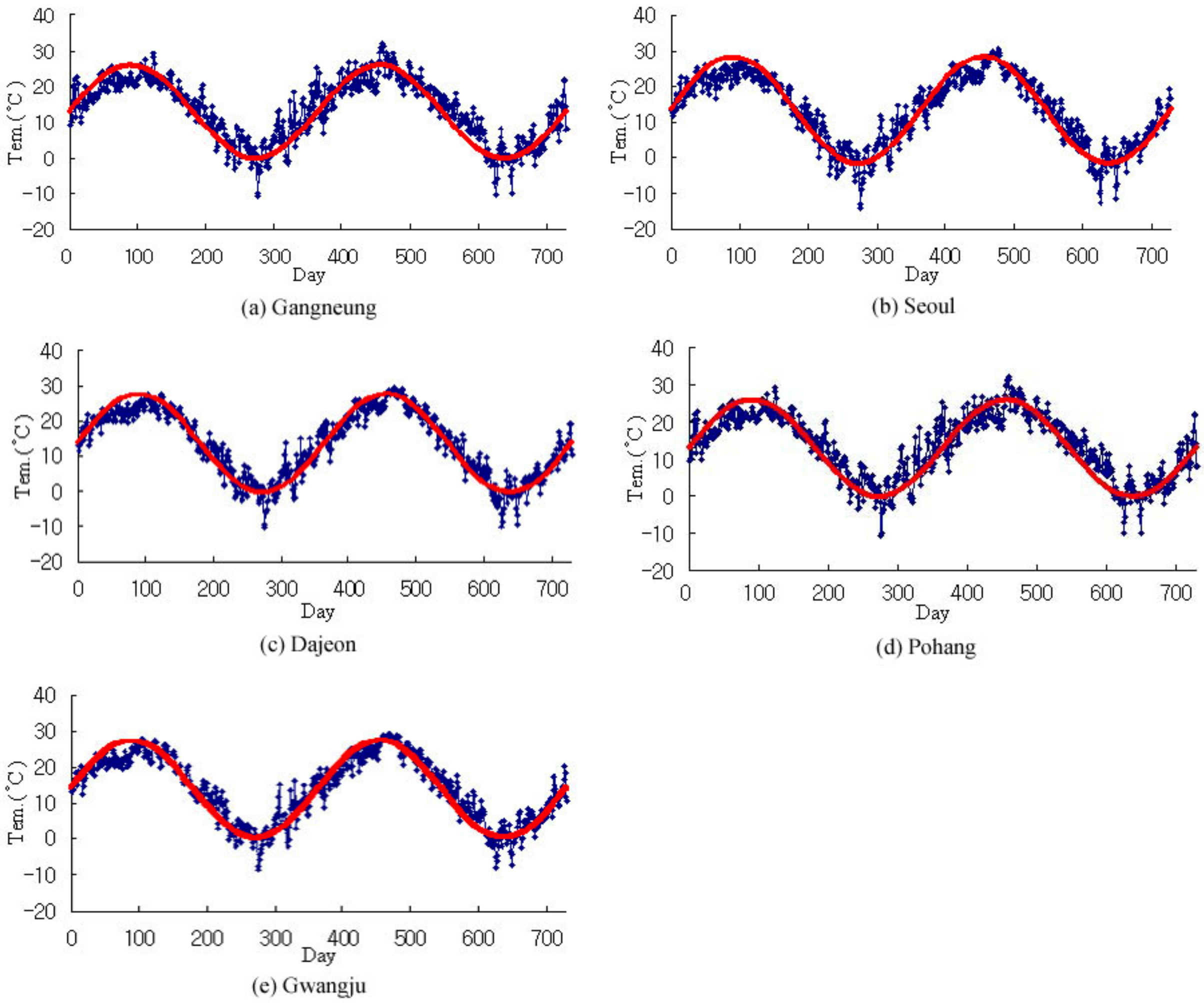


Figure 2. Temperature variations during recent two years at the 5 cities

Table 2. The maximum and minimum temperatures and the temperature deviations in the 5 cities during last two years

Area	Maximum ave. temperature(°C)	Year & month recorded	Minimum ave. temperature(°C)	Year & month recorded	Temperature deviation(°C)
Gangneung	24.5	2004.7	-0.4	2003.1	24.9
Seoul	26.1	2004.8	-2.5	2003.1	28.6
Dajeon	26.1	2004.7	-1.7	2003.1	27.8
Pohang	26.7	2004.7	1.2	2003.1	27.9
Gwangju	26.3	2004.8	0.2	2003.1	26.8

The solution for the first order thermal conductivity equation

We assume that the rock surface temperature is the same as the external temperature, and that the deep portions (deeper than ten meters) have a uniform temperature, which is widely acknowledged. Setting the depth of rock as L, the first order thermal conductivity equation can be expressed as follows:

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2},\quad \alpha^2=\frac{K}{c\rho}\qquad (0\le x\le L) \tag{2}$$

$$\begin{cases} u(0,t)=\varphi(t) \\ u(L,t)=u_0 \end{cases}\qquad (0\le x\le L)$$

$$\{u(x,0)=f(x)\qquad (0\le x\le L)$$

where *C* denotes specific heat, *K* denotes thermal conductivity coefficient, denotes density, denotes external temperature, and u_0 denotes the uniform temperature at *L*. The general solution for the equation can be obtained as follows:

$$u(x,t)=A(t)\left(1-\frac{x}{L}\right)+B(t)\frac{x}{L}+v(x,t) \tag{3}$$

After changing the variables, the original partial differential equation, and the boundary and the initial conditions become:

$$\frac{\partial v}{\partial t}=\alpha^2\frac{\partial^2 v}{\partial x^2}-\Psi(x,t)$$

$$\Psi(x,t)=\frac{d\varphi}{dt}\left(1-\frac{x}{L}\right) \tag{4}$$

$$v(0,t)=v(L,t)=0$$
$$v(x,0)=f(x)-s(x,0)=F(x)$$

The non-homogeneous differential equation and the homogeneous boundary value problem can be solved by the characteristic function expansion method.

The corresponding homogeneous differential equation:

$$\frac{\partial U}{\partial t}=\alpha^2\frac{\partial^2 U}{\partial x^2} \tag{5}$$

$$U(0,t)=U(L,t)=0$$

where, the characteristic function can be obtained by solving. Entering $U(x,t) = X(x)T(t)$and separating the variables as $X(x)$ is a function of x and $T(t)$ is a function of t yield.

$$\begin{cases} T'-\lambda^2\alpha^2 T=0 \\ X''-\lambda^2 X=0 \end{cases} \tag{6}$$

From the above equations, the characteristic function $X_n(x)$ becomes:

$$X_n(x)=\sin\left(\frac{n\pi x}{L}\right),\left(\lambda=\frac{n\pi}{L},n=1,2,\cdots\right) \tag{7}$$

Series expansion of (7) with respect to the second equation in (4) yields:

$$\begin{aligned}\Psi(x,t)&=\sum_{k=1}^{n}\hat{\Psi}_k(t)X_k(x)=\sum_{k=1}^{n}\hat{\Psi}(t)\sin\left(\frac{k\pi x}{L}\right)\\ &=\sum_{k=1}^{n}\frac{2}{k\pi}\varphi'(t)\sin\left(\frac{k\pi x}{L}\right)\end{aligned} \tag{8}$$

Now the unknown function $v(x,t)$can be done similarly as follows:

$$\sum_{k=1}^{n}\left\{T_k'(t)+\left(\frac{\alpha k\pi}{L}\right)^2 T_k(t)+\hat{\Psi}(t)\right\}X_k(x)=0 \tag{9}$$

$$\sum_{k=1}^{n}T_k(0)X_k(x)=F(x) \tag{10}$$

where each $T_k(t)$ can be obtained by solving the initial value problem:

$$T_k'(t)+\left(\frac{\alpha k\pi}{L}\right)^2 T_k(t)=-\frac{2}{k\pi}\varphi'(t) \tag{11}$$

$$T_k(0)=\frac{2}{L}\int_0^L F(x)\sin\left(\frac{k\pi}{L}x\right)dx \tag{12}$$

Entering:

$$T_k(t)=C(t)e^{-\beta^2 t},\beta=\frac{\alpha k\pi}{L} \tag{13}$$

into (11) determines

$$C(t)=-\frac{2}{k\pi}\int_0^t e^{\beta^2 s}\varphi'(s)ds+d \tag{14}$$

Therefore the general solution for $T_k(t)$ is:

$$T_k(t)=C(t)e^{-\beta^2 t}=de^{-\beta^2 t}-\frac{2}{k\pi}\int_0^t e^{-\beta^2(t-s)}\varphi'(s)ds \tag{15}$$

where, the integer d can be determined as follows:

$$\begin{aligned}&d=\{\varphi(0)-u_0\}\frac{2}{L}\int_0^L\sin\left(\frac{k\pi}{L}x\right)dx+\frac{2}{L}\\&\int_0^L f(x)\sin\left(\frac{k\pi}{L}x\right)dx-\varphi(0)\frac{2}{L}\int_0^L\sin\left(\frac{k\pi}{L}x\right)dx\end{aligned} \tag{16}$$

Thus $v(x,t)$ is:

$$v(x,t)=\sum_{k=1}^{n}T_k(t)\sin\left(\frac{k\pi}{L}x\right)dx$$

$$\sum_{k=1}^{n}\left[\begin{aligned}&f(x)e^{-\beta_t^2 t}\\&+\frac{2}{k\pi}\left\{(-1)^k u_0-\varphi(0)\right\}e^{-\beta_k^2 t}\sin\left(\frac{k\pi x}{L}\right)\\&-\frac{2}{k\pi}\left\{\int_0^t e^{-\beta_k^2(t-s)}\varphi'(s)ds\right\}\sin\left(\frac{k\pi x}{L}\right)\end{aligned}\right] \tag{17}$$

and we can find the solution $u(x,t)$ using $v(x,t)$ and the following equation:

$$u(x,t)=\varphi(t)\left(1-\frac{x}{L}\right)+u_0\frac{x}{L}+v(x,t) \tag{18}$$

To examine the difference of depth under the influence of deterioration for each rock type, over time, considering the temperature, we conducted an analysis on sandstone, granite, and gneiss, which represent sedimentary rock, igneous rock, and metamorphic rock, respectively.

RESULTS

Thermal characteristics of the rocks

In the calculation of the depth of rocks under the influence of deterioration due to temperature changes, we conducted an analysis for sandstone, granite, and gneiss in the present study, where the physical properties of the rocks are listed in Table 3 (from Lee 1999) and the specific heat was set the same. Among those values, thermal conductivity coefficients of sandstone and granite were calculated using the two values, i.e., the maximum and the minimum, and that of gneiss was calculated using a single value.

Generally, the heat Q that passes through the cross section of an arbitrary area A is proportional to the area A, temperature difference T_1 T_2, and time t, and is inversely proportional to the thickness (length) x. Generally, smaller porosity increases the thermal conductivity of a rock. This can be expressed as:

$$Q = KAt(T_1 - T_2)/x \tag{19}$$

where, K denotes thermal conductivity coefficient.

Table 3. Thermal properties of the rocks used in this study

Rock type	**Coefficient of thermal conductivity, K (cal/m×day×°C)**		**Specific heat (cal/kg×°C)**	**Density (kg×m^3)**
	Max.	Min.		
Sandstone	43,200	60,000	180	2,670
Granite	36,000	55,200	180	2,650
Gneiss	45,600		180	2,700

Calculating the depth under the influence of freeze thawing

The rock surface can be assumed to have the same temperature as the external temperature, as it is in contact with the atmosphere. In addition, rock is known to have a uniform temperature at deep enough depths. The portions of rock slopes under the influence of freeze thawing were calculated using the first order thermal conductivity equation (1) that was obtained under aforementioned assumptions. Since the initial temperature distribution of the rock that is to be used in our calculation is unknown, we calculated the temperature distribution during a certain time period after internal temperature distribution reached a normal state over enough time in setting the initial condition.

Regional temperature distributions that are used in the calculations are as listed in Table 1; we obtained the temperature distribution curves for 0 month, 1 month, 5 months, and 9 months afterwards, and the crossing point of all the curves was set as the rock depth under the influence of external temperature. Those values are listed in Table 4; the result for Seoul is as shown in Figure 4. The calculation shows that the deterioration limit depth with regard to external temperature in Korea is approximately 8.4 ~10.7 m, and overall, thermal conductivity coefficient tends to be proportional to deterioration depth as shown in Figure 3.

Table 4. Deterioration depths influenced by temperature in the 5 cities

Rock type		**Sandstone**		**Granite**		**Gneiss**
Coefficient of thermal conductivity (cal/m×day×°C)		43,200	60,000	36,000	55,200	45,600
Deterioration depth (m)	Gangneung	8.5	9.6	8.4	9.9	9.1
	Seoul	9.0	10.7	9.1	9.9	9.1
	Daejeon	9.3	10.1	8.6	9.5	9.2
	Pohang	8.8	9.9	8.4	9.7	9.2
	Gwangju	9.0	9.5	8.6	10.0	8.8

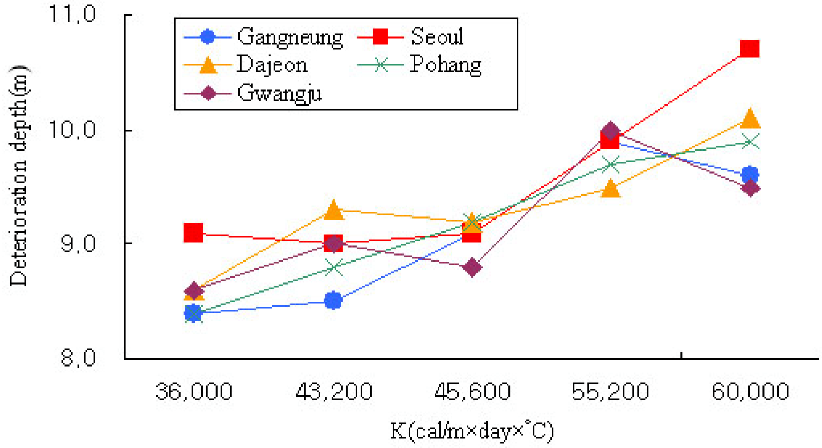
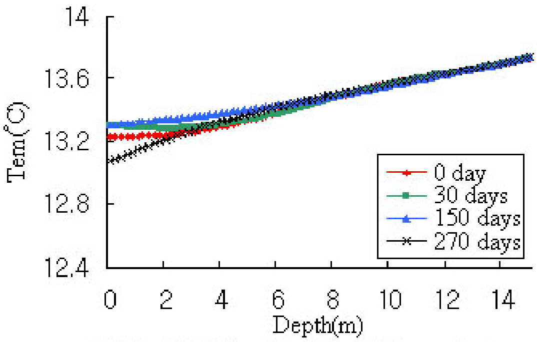
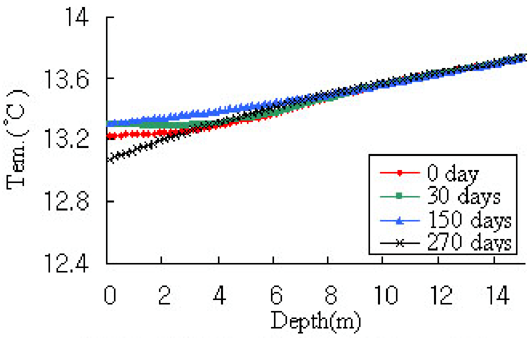


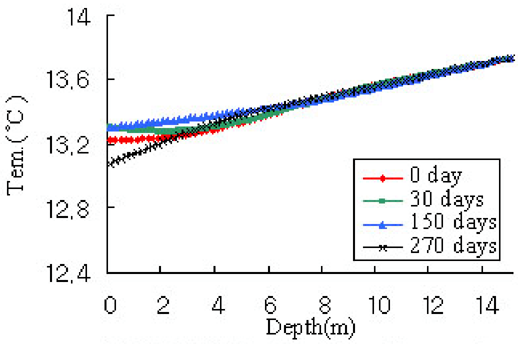
Figure 3. The relationship between thermal conductivity coefficient and the depths influenced by deterioration



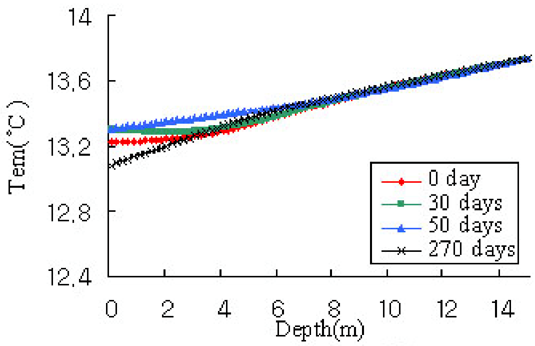
(a) K=43,200 cal/mxdayx˚C, sandstone



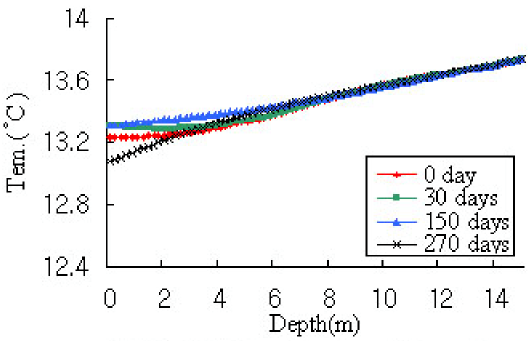
(b) K=60,000 cal/mxdayx˚C, sandstone



(c) K=36,000 cal/mxdayx˚C, granite



(d) K=55,200 cal/mxdayx˚C, granite



(e) K=45,600 cal/mxdayx˚C, gneiss

Figure 4. Temperature vs. depth with the elapsed time at Seoul

Regional deterioration characteristics

The results of analyzing the depths under the influence of rock deterioration in consideration of the thermal conductivity coefficient for each rock type in each area in Korea are as shown in Figure 5. Seoul, Daejeon, and Pohang areas exhibited greater depths under the influence of rock deterioration due to the external temperature regardless of rock type, as the thermal conductivity coefficient becomes greater. Among all the rocks that were under consideration, the largest thermal conductivity coefficient was found in sandstone, which is 60,000 cal/m×day×°C. If the rock is present in Seoul, up to 10.7 m; for Daejeon up to 10.1 m; for Pohang up to 9.9 m depths are estimated to be under the influence of freeze thawing due to external temperature. Seoul, Daejeon, and Pohang, where the thermal conductivity coefficient is proportional to the rock depth under the influence of deterioration regardless of rock type, correspond to the areas where the difference between the highest and the lowest temperatures for the past two years is the greatest, among the five areas that are considered in the present study. The type of rock with the smallest depth under the influence of deterioration in those three areas is granite with the thermal conductivity coefficient of 36,000 cal/m×day×°C estimated depths under the influence of deterioration are 9.1 m for Seoul, 8.6 m for Daejeon, and 8.4 m for Pohang.

Among the five areas Gwangju and Gangneung show a different result from the three aforementioned areas. In Gwangju, the type of rock that exhibits the greatest depth under the influence of deterioration is granite, with the second largest thermal conductivity coefficient of 55,200 cal/m×day×°C, of which the influenced depth is estimated 10.0 m; that is approximately 0.5 m deeper than when sandstone is present, which has the greatest thermal conductivity coefficient.

Gangneung, which shows the smallest temperature variation of 24.9 °C in the five areas, is estimated to have the greatest depth under the influence of deterioration, i.e., 9.9 m, when granite of thermal conductivity coefficient 55,200 cal/m×day×°C is present; that is approximately 0.3 m deeper than when sandstone is present, which has the greatest thermal conductivity coefficient.

When the estimated depth under the influence of rock deterioration is the greatest for the area, different results were obtained from large temperature variation and small temperature variation yet when the estimated depth of influence is low for the area, smaller thermal conductivity coefficients coincided with smaller influenced depth of rock regardless of temperature variation in all areas.

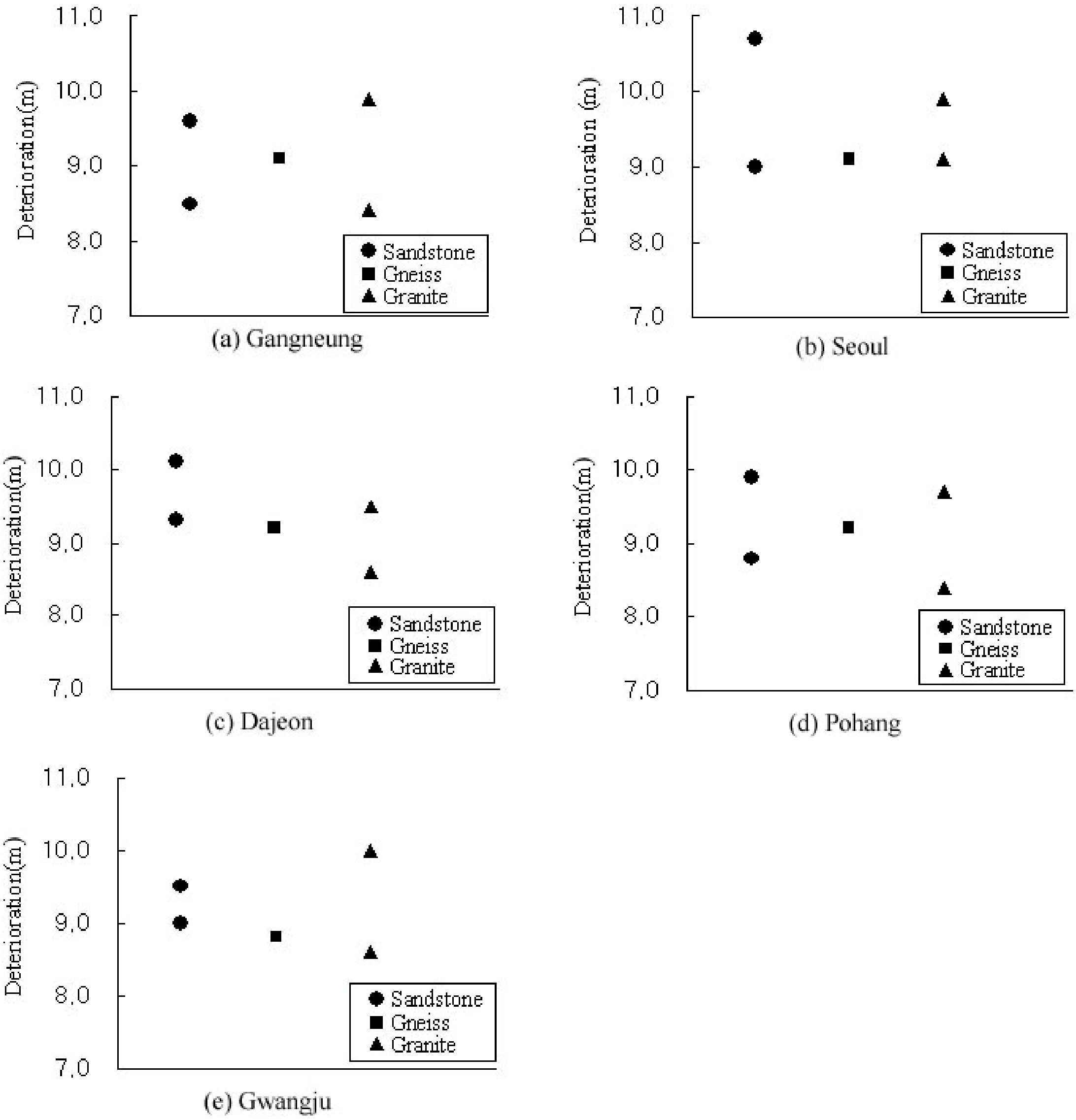


Figure 5. Deterioration depths at the 5 cities

Deterioration characteristic for each rock type

The results of analyzing the rock depth under the influence of rock deterioration for each rock type are as shown in Figure 6. When the sandstone of the K value 43,200 cal/m×day×°C is present, Daejeon is estimated to have the greatest depth of influence and Gangneung is estimated to have the least depth of influence; the depths are 9.3 m and 8.5 m, respectively. Sandstone, which has the greatest thermal conductivity coefficient of 60,000 cal/m×day×°C among all the types of rocks, has its greatest depth of influence in Seoul, i.e., 10.7 m; its smallest depth of influence is found in Gwangju, which is 1.2 m smaller than that in Seoul. When gneiss is present in each area, Daejeon and Pohang show the same greatest value of 9.2 m, whereas Gwangju shows the smallest value of 8.8 m. Granite with the thermal conductivity coefficient of 36,000 cal/m×day×°C, has its greatest depth under the influence of rock deterioration of9.1 m in Seoul, which is 0.7 m greater than the smallest depth of influence that is estimated in Gangneung and Pohang. When granite with K of 55,200 cal/m×day×°C is present, Gwangju's freeze thawing has the greatest influence of depth on rock and Deajeon is estimated to have the smallest value the depths are 10 m and 9.5 m, respectively.

Among all the rock types, the one that has the greatest depth of influence is sandstone with thermal conductivity coefficient of 60,000 cal/m×day×°C, of which the depth is 10.7 m and the area is Seoul, which has the greatest difference between the highest and the lowest temperatures.

Analysis for each rock type excluding granite with thermal conductivity coefficient of 55,200 cal/m×day×°C shows that the type of rock in the area of greater temperature variation has the greater depth under the influence of rock deterioration. When granite of thermal conductivity 55,200 cal/m×day×°C is present, estimated depth of influence was greater in Gwangju and Gangneung, which have smaller temperature variation in the five areas, than in the areas with the greater temperature variation.

Figure 6. Deterioration depths of each rock type

CONCLUSION

Using the temperature distributions over the past two years for five major cities in Korea, we calculated the depths under the influence of rock deterioration for sandstone, granite, and gneiss, which occurs substantially in Korea. Deterioration depth due to freeze thawing in Korea is approximately 8.4 10.7 m, which shows that overall, thermal conductivity coefficient is proportional to deterioration depth.

According to the regional analysis, among the five areas considered in the present study, the areas with great difference between the highest and the lowest temperatures, i.e., Seoul, Daejeon, and Pohang, are estimated to have a tendency in which the thermal conductivity coefficient is proportional to the rock depth under the influence of rock deterioration. Areas with smaller temperature variations, i.e., Gwangju and Gangneung, exhibit different results than the three aforementioned areas; the rock type with the greatest depth of influence in Gwangju and Gangneung is granite, with the second largest thermal conductivity coefficient of 55,200 cal/m×day×°C.

The analysis by rock type estimates that when granite with thermal conductivity coefficient of 55,200 cal/m×day×°C is present, Gwangju and Gangneung that have relatively small temperature variation among the five areas have greater depths of influence than the areas with greater temperature variations. Excluding the granite, rocks that are present in areas with greater temperature variations tend to have greater depths under the influence of rock deterioration.

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