Predicting the shear strength parameters of mudrocks

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Abstract: Shear strength is one of the most important properties for design of engineering structures built on, or within, mudrocks (shales, claystones, mudstones, siltstones, etc.), and also one of the most difficult to evaluate. This is because it is usually difficult to obtain undisturbed samples of mudrocks required for determination of shear strength parameters. The purpose of this research was to investigate the geological and engineering properties of mudrocks that can be used statistically to predict the shear strength parameters of a broad range of mudrocks.

Forty-five samples of various types of mudrock were collected from highway cuts throughout the United States. Clay content, clay mineralogy, water content, Atterberg limits, specific gravity, dry density, void ratio, absorption, adsorption, slake durability, and shear strength parameters (c and φ) were determined for each sample. Data were analyzed statistically, using bivariate and multiple regression techniques, to determine the correlations between shear strength parameters and geological and engineering properties. Based on the statistical analyses, prediction models were developed for all mudrock types as a single group as well as separately for shales, claystones, mudstones, and siltstones.

Preliminary results show that cohesion and friction angle parameters for all mudrocks, treated as one population, can be predicted from selected properties (amount and type of clay, Atterberg limits, % absorption, % adsorption, dry density, void ratio, and slake durability) with adjusted coefficient of regression, r^2, values of 0.77 and 0.81, respectively. When mudrocks are subdivided into individual types and analyzed separately, the adjusted r^2 values for predictive models are improved. The cohesion for shales, claystones, mudstones, and siltstones can be predicted with r^2 values of 0.85, 0.98, 0.99, and 0.98, respectively, whereas the r^2 values with respect to friction angle are found to be close to 0.99 for all four types of mudrock.

Résumé: Quarante-cinq échantillons de divers types de roches argileuses ('claystones', 'mudstones', 'siltstones', et 'shales', c'est à dire, respectivement non-laminés et >50% la taille d'argile, 33-49%, 0-32%, et schistes argileux laminés), sont collectés partout aux Etats-Unis, pour une série des analyses de caractéristiques lithologiques et de propriétés techniques. Les essais ont été analysés statistiquement, en utilisant les techniques bivariate et multivariables de régression, pour déterminer les corrélations entre les paramètres de résistance au cisaillement (la cohésion et l'angle de frottement), les caractéristiques lithologiques, et les propriétés techniques. D'après les analyses statistiques, les modèles de prévision ont été développés, pour la cohésion et le frottement, d'abord pour tous les types de roches argileuses en tant qu'un seul groupe et puis pour chaque type de roche individuelle (c'est à dire, 'claystones', 'mudstones', 'siltstones', et 'shales'). Les résultats prouvent que les paramètres de la cohésion et de l'angle de frottement pour tous les roches argileuses, traités en tant qu'une population, peuvent être prévues selon quelques propriétés specifiques (la quantité et le type d'argile, les limites d'Atterberg, % d'absorption, % d'adsorption, la densité sèche, le rapport des vides, et 'slake durability') avec les valeurs r^2 (ajustées) de 0.77 et de 0.81, respectivement. Quand les roches argileuses sont subdivisés en différents types et sont analyses séparément, les valeurs ajustées de r^2 pour les modèles prédictifs sont améliorées. La cohésion pour les 'claystones', les 'mudstones', les 'siltstones', et les 'shales' peut être prévue avec les valeurs r^2 de 0.98, de 0.99, de 0.98, et de 0.85, respectivement, tandis que les valeurs R^2 en ce qui concerne l'angle de frottement sont près de 0.99 pour chacun des quatre types de roches argileuses.

Keywords: shear strength, cohesion, friction, engineering properties, clay minerals, data analysis.

INTRODUCTION

The term mudrock (claystones, mudstones, siltstones, shales) refers to the fine-grained, siliciclastic sedimentary rocks in which more than 50% of the particles are smaller than 0.06 mm in size (Blatt Middleton & Murray., 1980, Grainger, 1984; Dick & Shakoor, 1992). Mudrocks constitute about 45% to 55% of sedimentary rock sequences; thus they are often encountered in engineering construction. Information about shear strength parameters (cohesion, c, and friction angle, ϕ) is frequently required for design and stability analysis of engineering structures located on, or within, mudrocks. However, it is often difficult to obtain good quality, undisturbed samples of mudrocks for shear testing because they are weak and sensitive to changes in moisture, drilling pressure, and time (Fam, Dussealt & Fooks, 2003) Therefore, information about shear strength parameters for most engineering projects involving mudrocks is either scarce or assumed. One way to overcome this problem is to develop a methodology that could be used to predict the shear strength parameters of mudrocks from their lithological characteristics (clay content, clay mineralogy) and other engineering properties (natural water content, specific gravity, void ratio, absorption, adsorption, Atterberg limits, slake durability), which are relatively easy to determine. Although some research has been done to relate durability, swelling potential, and unconfined compressive strength of mudrocks to their lithological characteristics and engineering properties (Dick, Shakoor & Wells, 1994; Sarman, Shakoor & Palmer, 1994; Greene, 2001), very little research has been conducted to investigate the relationships between shear strength parameters and other properties of mudrocks. A study by Olgaard et al. (1997) found that shear strength of mudrocks was inversely proportional to the swelling clay content, suggesting the need for further research in this area.

PURPOSE OF STUDY

The purpose of this study is to investigate whether or not the strength parameters of mudrocks can be predicted from their lithological characteristics and other engineering properties. If meaningful correlations are found to exist between shear strength parameters, lithological characteristics, and engineering properties of mudrocks, these could be used to estimate cohesion and friction angle in situations where either the good quality samples required for shear testing or the equipment needed to conduct such tests are not available.

RESEARCH METHODS

Sampling

Forty-five mudrock samples were collected from various parts of the United States and classified according to the system shown in Table 1. The samples include 10 claystones, 10 mudstones, 12 siltstones, and 13 shales (8 siltshales, 4 mudshales, 1 clayshale). In order to minimize the effect of weathering on shear strength of mudrock samples, maximum care was taken to assure that the collected samples were as fresh as possible.

Table 1. Geological classification of mudrocks (modified after Potter Maynard & Pryor, 1980).

	Percent Clay-Size Particles		
	0 – 32 %	33 – 49 %	50 – 100 %
Nonlaminated	Siltstone	Mudstone	Claystone
Laminated	Siltshale	Mudshale	Clayshale
Metamorphosed	Argillite		

Laboratory Investigations

A series of laboratory tests were performed to determine the lithological characteristics and engineering properties of mudrocks sampled in accordance with the standard procedures of the American Society for Testing and Materials (ASTM, 1996), where applicable. Samples were pulverized for some of the tests using repeated freeze-thaw cycles. Three tests were performed for determination of each property and results were reported as the average of three tests. Lithological characteristics investigated included the amount of clay size material (< 0.004 mm and < 0.002mm), as determined by hydrometer analysis, and the type and amount clay minerals, as determined by x-ray diffraction analysis. Engineering properties investigated included natural water content, specific gravity, void ratio, absorption, adsorption, Atterberg limits (liquid limit, plastic limit, plasticity index), slake durability, and shear strength parameters, cohesion and friction angle. The strength parameters were determined by the direct shear test method.

Table 2 summarizes the cohesion values for mudrocks and Figure 1 shows the distribution of these values for different mudrock groups. The cohesion values show a wide range from a minimum of 5095.7 psf (0.24 MPa) to a maximum of 163857.6 psf (7.85 MPa), with the shale group exhibiting the maximum variability. This is not unexpected as this group includes the siltyshale, mudshale, and clayshale samples.

The friction angle values for various mudrock groups are summarized in Table 3 and the distribution of these values is presented graphically in Figure 2. Like cohesion, the values of friction angle for all mudrocks represent a broad range from a minimum of 10.1° to a maximum of 34.8°, with claystones having largest variation and siltstones the smallest.

Table 2. Summary of cohesion values for all mudrocks and lithological subgroups.

Lithologic Group	**N**	**Minimum (psf)**	**Maximum (psf)**	**Mean (psf)**	**Variance (psf)**	**Standard Deviation (psf)**
All Mudrocks	45	5,095.7	163,857.6	52,141.0	1,588,555,488.8	39,856.7
Claystones	10	7,071.6	65,991.5	30,334.9	372,221,323.2	19,293.0
Mudstones	10	13,859.4	78,261.1	47,310.3	531,804,880.0	23,061.0
Siltstones	12	10,748.9	132,302.9	67412.8	1,456,309,926.9	38,161.6
Shales	13	5,095.7	163,857.6	58,533.7	2,913,776,286.2	53,979.4

Note: 1 psf = 4.79 x 10^{-5} MPa.

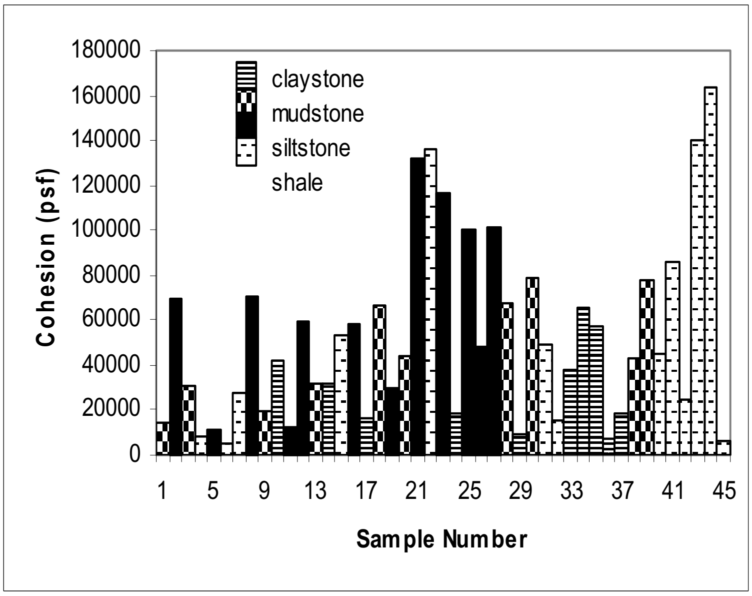

Figure 1. Distribution of cohesion values for various mudrock groups.

Table 4 summarizes the data for other properties of mudrocks including maximum, minimum, and mean values of each property as well as the values of variance and standard deviation. Data for claystones, mudstones, siltstones and shales were also tabulated in a similar manner.

Table 3. Summary of friction angle values for all mudrocks and lithological subgroups.

Lithological Group	**N**	**Minimum (°)**	**Maximum (°)**	**Mean (°)**	**Variance (°)**	Standard deviation (°)
All Mudrock	45	10.9	35.8	24.9	60.1	7.8
Claystone	10	10.9	30.9	22.4	59.2	7.7
Mudstone	10	13.2	29.6	19.1	33.3	5.8
Siltstone	12	22.0	35.8	32.4	15.3	3.9
Shale	13	13.8	34.3	24.4	41.4	6.4

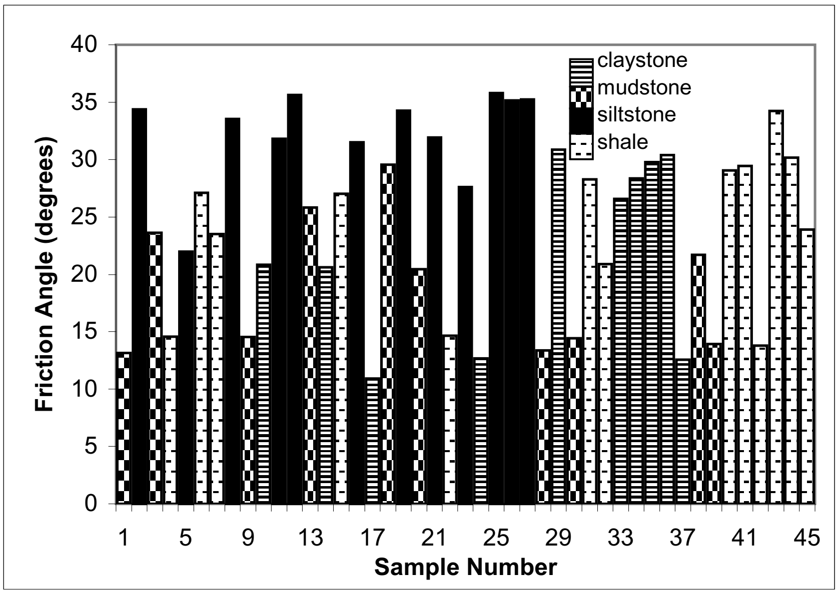

Figure 2. Distribution of friction angle values for various mudrock groups.

Table 4. Summary of lithological characteristics and engineering properties for all mudrocks.

Variables	Statistical Parameters				
	Minimum	Maximum	Mean	Variance	Standard Deviation
< 0.002 mm Clay (%)	12	77	28.2	163.7	12.8
< 0.004 mm Clay (%)	18	82	37.1	213.4	14.6
Chlorite (%)	0	2.6	0.5	0.4	0.7
Kaolinite (%)	1.1	53.1	12.2	108.2	10.4
Illite (%)	0	21	8.5	26.3	5.1
Mixed-Layer Clay (%)	0	32.3	6.1	52.1	7.2
Montmorillonite (%)	0	14.6	0.8	7.3	2.7
Expandable Clay (%)	0	47	6.9	75.7	8.7
Non-expandable Clay (%)	2.3	56.5	21.2	112	10.6
Expand. / Non-expand. Clay	0	5	0.6	1.2	1.1
Water Content (%)	0.5	22.9	4.5	24.6	5
Dry Density (lb/ft^3)	119.2	181.7	150.3	138.6	11.8
Specific Gravity	2.5	3	2.7	0.008	0.1
Void Ratio	0.01	0.4	0.1	0.006	0.08
Absorption (%)	0.8	83.7	17.9	432.9	20.8
Adsorption (%)	0.2	11.7	3.3	6.5	2.6
Liquid Limit	17.4	75	30.3	135.9	11.7
Plastic Limit	14	30.2	20.5	18.1	4.3
Plasticity Index	0.5	45.2	9.8	86.2	9.3
Slake Durability Index (%)	1	99.1	71.4	1201	34.7
Cohesion (lb/ft^2)	5095.7	163857.6	52141	1.59E+09	39857
Friction Angle (^{o})	10.9	35.8	24.9	60.1	7.8

Note: 1 psf = 4.79 * 10^{-5} MPa

Data Analysis

Data from geological and engineering tests were compiled and statistically analyzed to investigate the predictability of cohesion and friction angle from lithological characteristics and engineering properties. Data produced from three tests for each property were all used in the statistical analysis to compensate for geological variations within the individual mudrock samples and to increase the number of observations. Univariate, bivariate, and multivariate analyses were first preformed for all mudrocks samples treated as a one group and then for individual subgroups. Two more independent variables were added to the analyses: lithology (1 for siltstone, 2 for mudstone, 3 for claystone, and 4 for shale) and lamination (1 for laminated rock and 0 for non-laminated rock). The statistical analyses were preformed using an interactive statgraphics personal computer program SPSS 12.0 for windows (SPSS, 2003).

RESULTS OF STATISTACAL ANALYSIS

Univariate Analysis

Results from univariate analysis were used to describe the distributional characteristics of various properties (variables) tested. In general, about half of the variables (percent clay <0.004 mm, percent illite, percent non-expandable clay, dry density, specific gravity, void ratio, percent absorption, percent adsorption, plastic limit, and slake durability index) approximate normal distribution, with their standardized skewness ‘Ss’ and standardized kurtosis ‘Ks’ falling within the range of +2 to –2. The variables that did not meet the Ss and Ks criteria for normal distribution are percent clay <0.002 mm, percent chlorite, percent kaolinite, percent mixed-layer clay, percent montmorillonite, percent expandable clays, percent expandable to non-expandable clays, natural water content, liquid limit, and plasticity index. Logarithmic and square root transformations of most of these variables produced normal distributions.

Bivariate Analysis

Bivariate analysis was preformed to find out if cohesion and friction angle parameters showed statistically significant correlations with individual variables. The correlation coefficient (r) and the corresponding two-tailed t-test level of significance were determined. In general, none of the variables showed a statistically significant correlation ($r > 0.81$) with cohesion or friction angle. Cohesion values showed the strongest correlations with slake durability index and absorption, whereas friction angle values showed the strongest correlations with percent mixed layer clay, percent expandable clay, expandable to non-expandable clay ratio, natural water content, absorption, adsorption, and slake durability index.

Multivariate Regression Analysis

The lack of strong correlation between cohesion or friction angle values of mudrocks and any one of the other variables in bivariate analysis indicates that no single lithological characteristic or engineering property can provide a meaningful predictor of these strength parameters. Given the lithological diversity of mudrocks, it is most likely that cohesion and friction angle of mudrocks are controlled by a combination of variables. Therefore, multivariate regression analysis, using backward stepwise approach, was preformed on mudrock data to identify the set of variables that correlate best with cohesion and friction angle. Each model produced from each regression step was accompanied by an adjusted squared multiple correlation factor (r^2) and an estimated error of calculation.

The backward regression analysis of all mudrock data, treated as a single group, produced 29 models for prediction of cohesion. Figure 3 shows a plot of model number versus the adjusted r^2. From a practical point of view, the model selected should be the one that has the highest value of adjusted r^2, the least number of variables involved (i.e. the least number of engineering or lithological tests), and a maximum estimated error of approximately 20 % of the minimum cohesion value of the data used in the analysis. Based on these considerations, model number 14 was selected as the best model. This model has an adjusted r^2 of 0.774 and an estimated error of calculation of ±18,929.8 psf (0.91 MPa). This methodology of selection the best model was used for all cohesion and friction models in this study. The regression equation from model number 14 is as follows:

Cohesion (psf) = (16390.8 * Lithology) - (50001.8 * Lamination) - (919915.9 * Log % 0.002 mm Clay) - (11193.1* % 0.004 mm Clay) + (1308553.2 * Log % 0.004 mm Clay) - (55479.0 * % Chlorite) + (89927.2 * √% Chlorite) + (8742.4 * % Kaolinite) + (9004.5 * % Illite) + (6932.9 * % Mixed Layer Clay) + (27910.6 * √% Mixed Layer Clay) + (59156.1 * √% Montmorillonite) - (20005.2 * √% Expandable Clay) + (12696.8 * % Expandable Clay/Non-expandable Clay) + (4391.8 * Natural Water Content) - (18896.8 * √ Natural Water Content) - (4211.8 * Dry Density) - (3135509 * Specific Gravity) + (20579558 * Log Specific Gravity) - (532699.1 * Void Ratio) - (2872.2 * % Absorption) - (89098.2 * Log % Absorption) + (50920.8 * √% Absorption) + (4326. 9 * % Adsorption) - (916115.1 * Log Liquid Limit) + (541481.8 * Log Plastic Limit) - (163239.7 * Log Plasticity Index) + (105265.2 * √Plasticity Index) + (58753.5 * Log Slake Durability Index) + 57340.4 (1)

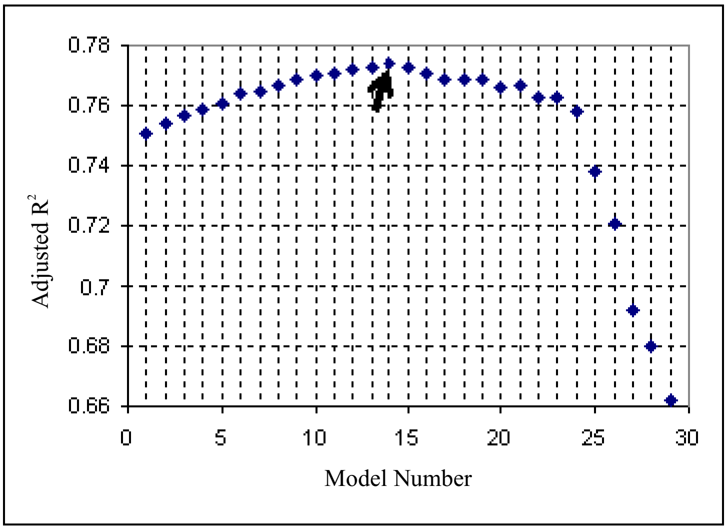

Figure 3. Plot of regression model number and corresponding adjusted r^2 values with respect to cohesion.

Equation 1 was used to predict cohesion values for all mudrocks. Figure 4 shows a plot of predicted (as determined from Equation 1) versus measured values of cohesion. The effectiveness of the prediction model is indicated by the proximity of the plotted points to the 1:1 (45°) line. The closer the points to the line, the more effective the model and the equation are. Examination of the data in Figure 4 shows that there are large differences between the measured and the predicted values of cohesion that reflect the model's large estimated error. Therefore, Equation 1 is considered to be an unacceptable predictor of mudrock cohesion values. The cohesion data must be investigated separately for the individual subgroups.

A similar multivariate analysis was performed to predict the friction angle for all mudrocks, yielding 38 models. The equation for the best model in this case had the following form:

Friction Angle (degrees) = (-11.24 * Lithology) + (23.34 * Lamination) - (45.26 * Log % 0.002 mm Clay) + (1.17 * % 0.004 mm Clay) + (3.11 * % Chlorite) - (0.36 * % Illite) - (0.62 * % Mixed Layer Clay) + (4.92 * √% Montmorillonite) + (0.94 * % Expandable Clay/Non-expandable Clay) + (1.86 * Natural Water Content) + (3.85 * √% Absorption) + (119.99 * Log Plastic Limit) - (9.24 * √% Chlorite) + (2.13 * √% Illite) + (5.40 * √% Mixed Layer Clay) - (0.88 * % Montmorillonite) - (7.08 * √% Expandable Clay) + (12.23 * Log Natural Water Content) - (13.86 * √Natural Water Content) + (0.53 * Dry Density) - (127.44 * Log Sp. Gravity) + (34.57 * √Void Ratio) - (0.33 * % Absorption) - (4.11 * √Adsorption) - (2.89 * Plastic Limit) + (0.83 * Plasticity Index) - (6.41 * √ Plasticity Index) - 37.84 (2)

Figure 5 shows a plot of measured versus predicted values (from Equation 2) of friction angle. Again there are large differences between measured and predicted values.

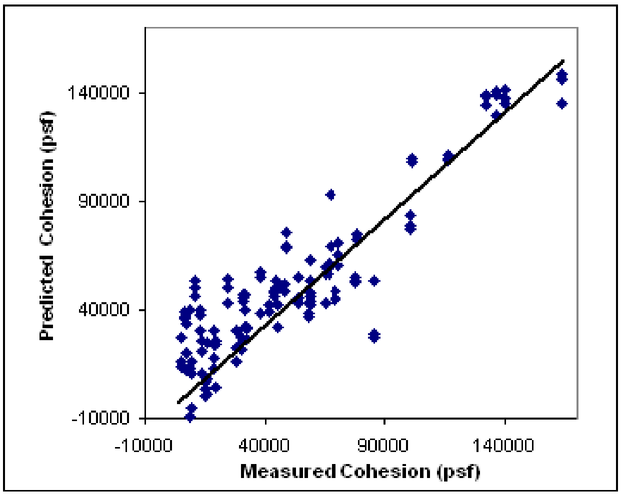

Figure 4. Measured versus predicted cohesion values for all mudrocks.

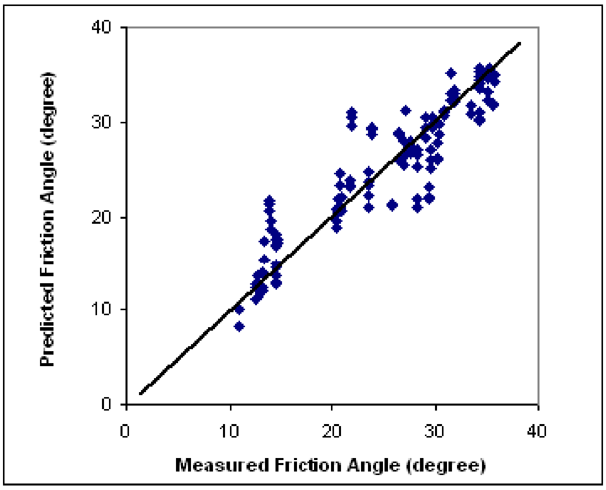

Figure 5. Measured versus predicted friction angle values for all mudrocks.

The next step in statistical analysis focused on investigating the predictability of cohesion and friction angle for each subgroup of mudrocks. Similar to the case of all mudrocks, logarithmic and square root transformations were applied to all those variables in the subgroups that were indicated by the univariate analysis not to be normally distributed. Bivariate analysis showed stronger correlations between cohesion or friction angle and other variables compared to all mudrocks treated as one group but no single variable exhibited statistically significant correlation to serve as a reliable predictor of cohesion or friction angle for any of the four subgroups. Therefore, backward multivariate regression analysis was used to develop prediction models on the basis of various combinations of variables. This resulted in development of 18 models for prediction of cohesion for claystones, with model number 9 being the best model. The adjusted R^2 value improved from a maximum of 0.77 in case of all mudrocks to 0.98 for claystones treated as a separate group, and the estimated error dropped dramatically to an acceptable value of 2397.6 psf (0.11 Mpa). The prediction equation, based on model number 9, is as follows:

Cohesion (psf) = (-6074391.0 * Log % 0.004 mm Clay) + (69939.79 * % Chlorite) + (33755.94 * % Kaolinite) + (26050.83 * % Illite) + (153925.66 * √% Mixed Layer Clay) + (65735.53 * %Montmorillonite) - (99198.33 * Log Natural Water Content) + (482969.06 * Log Specific Gravity) - (68136.47 * Void Ratio) + (15943.10 * Plastic Limit) - (869471.70 * Log Plastic Limit) + (2202.03 * Plasticity Index) - (41856.71 * Log Plasticity Index) - (19548.85 * Log Slake Durability Index) + 9809157.80 (3)

Equation 3 can be used to predict the cohesion of claystones providing the data for 14 variables that make the equation are available. These data can be obtained by performing seven different tests including hydrometer, x-ray diffraction, slake durability, Atterberg limits, specific gravity, water content, and void ratio tests. A plot of predicted (from Equation 3) versus measured values of cohesion for claystones is given in Figure 6. The data points in Figure 6 fall close to the 1:1 line indicating reliability of Equation 3 in predicting cohesion values with small amount of estimated error.

Backward regression analysis with respect to friction angle for claystones yielded 22 models. Model number 19 was chosen as the best practical model to predict friction angle, as it required only one test (x-ray diffraction). The model has an adjusted r^2 of 0.98 and an estimated error of ± 1.04 degrees, an improvement from the friction angle model for all mudrocks treated as a single group. The regression equation for friction angle of claystones can be written as:

Friction Angle (degrees) = (4.54 * % Chlorite) + (0.56 * % Kaolinite) + (0.19 * % Illite) - (0.88 * √% Mixed Layer Clay) + 4.54 (4)

A plot of measured versus predicted values of friction angle is given in Figure 7. The plot indicates a close agreement between the measured and predicted values (maximum difference of 1.5 to 2.0 degrees).

Similar equations and plots were developed for the other mudrock subgroups using multivariate analysis. Figures 8 through 13 show the predicted versus measured values of cohesion and friction angle for mudstones, siltstones, and shales, respectively. The adjusted r^2 values for cohesion plots for mudstones, siltstones, and shales are 0.99, 0.98, and 0.85, respectively, whereas friction angle plots have r^2 values close to 0.99.

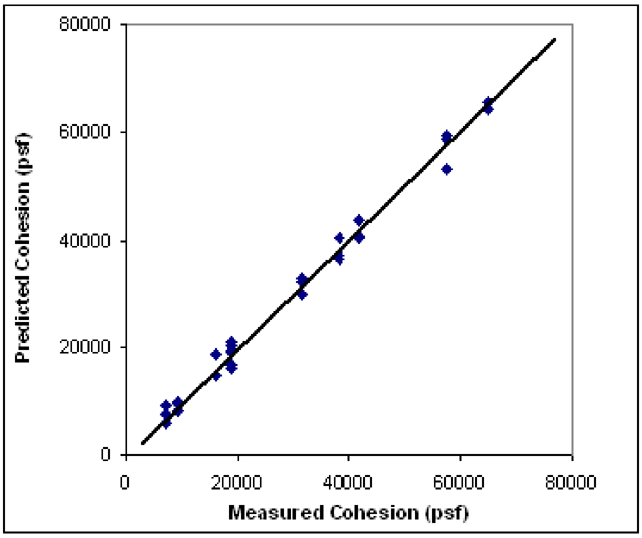

Figure 6. Measured versus predicted values of cohesion for claystones.

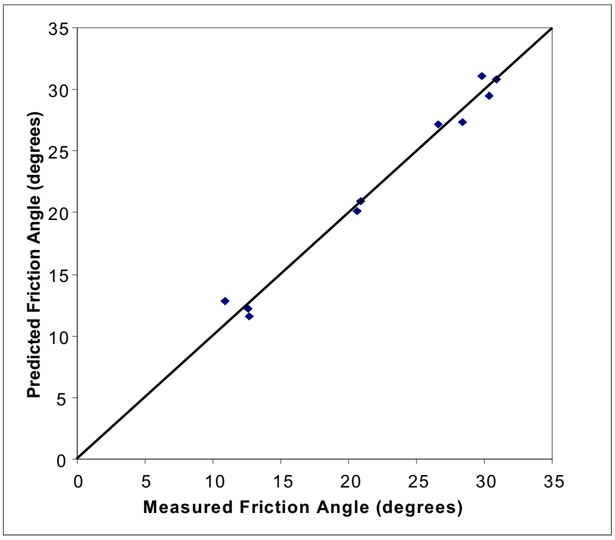

Figure 7. Measured versus predicted values of friction angle for claystones.

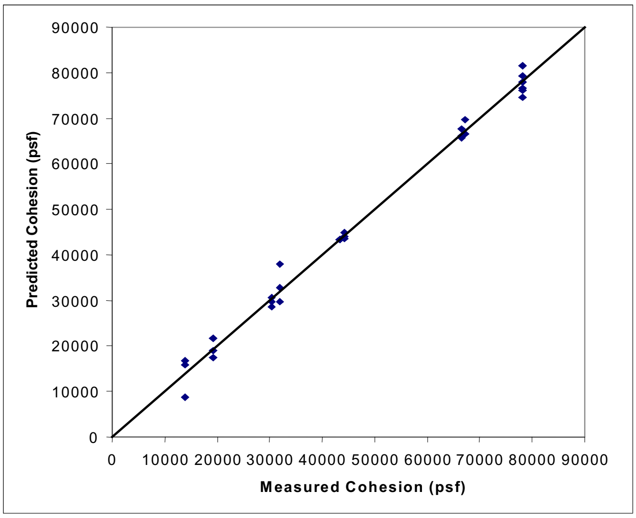

Figure 8. Measured versus predicted values of cohesion for mudstones.

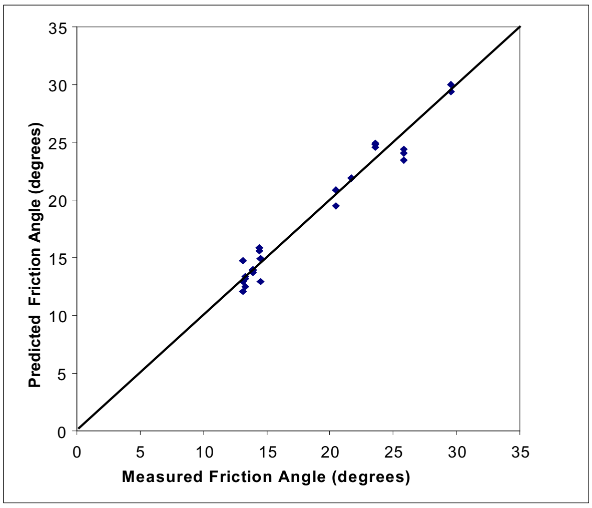

Figure 9. Measured versus predicted values of friction angle for mudstones.

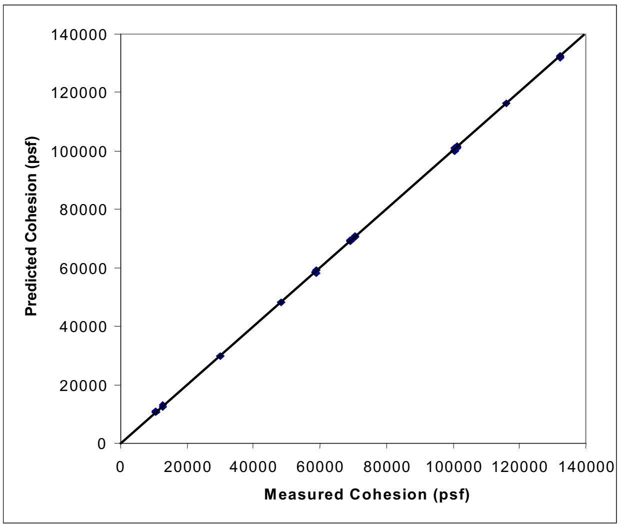

Figure 10. Measured versus predicted values of cohesion for siltstones.

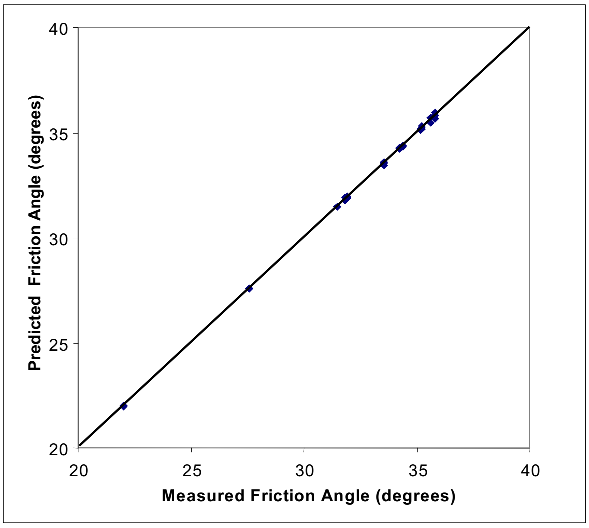

Figure 11. Measured versus predicted values of friction angle for siltstones.

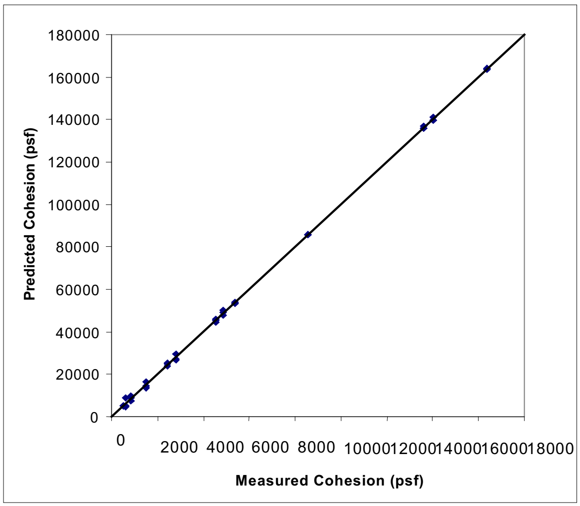

Figure 12. Measured versus predicted values of cohesion for shales.

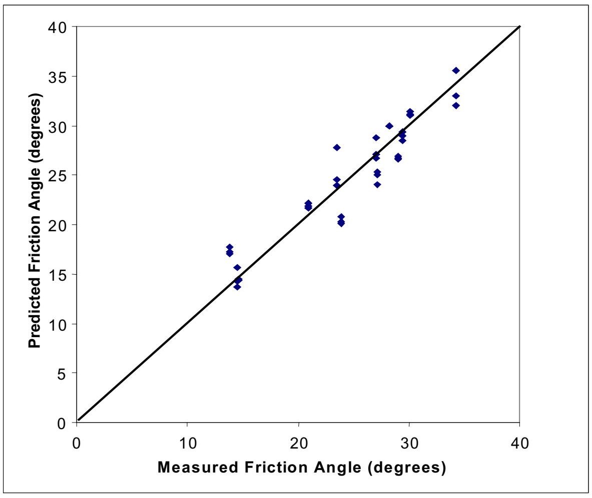

Figure 13. Measured versus predicted values of friction angle for shales.

CONCLUSIONS

It can be concluded on the basis of this study that no single geological or engineering property can be used to predict shear strength parameters of all mudrocks because of their highly variable nature. However, dividing mudrocks into subgroups (claystones, mudstones, siltstones, shales) reduces the lithological diversity and allows establishment of predictive equations that can be used to empirically estimate cohesion and friction angle values from the lithological characteristics and engineering properties of subgroups. The predictive equations developed in this study can be used to estimate shear strength parameters when either the good quality samples for shear testing or the required equipment are not available.

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