Rheological FEM numerical analyses for deformation mechanism of the revival old landslide with interbedded sandstone and claystone

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Abstract: Based on the analyses of time-dependent propagation mechanism of old landslides deformation, a hybrid rheological model is presented and creep and relaxation constitutive equations are derived by means of Laplace transformation. A visco-elastic plastic FEM program based on the hybrid rheological model is developed and applied to the rheological numerical analysis of an old landslide located in Wanxian. The results are compared with geomechanics analysis and macro-geology phenomena, and prove that the presented analysis method is an effective approach to landslide mechanism analysis.

Résumé: Basé sur les analyses pour le mécanisme se développant dépendant du temps de la vieille déformation d'éboulements, un modèle rhéologique hybride est présenté et des équations constitutives de fluage et de relaxation sont dérivées au moyen de transformation de Laplace, qui sur l'état initial de t=0. Un programme visco-élastique du plastique FEM sur la base du modèle rhéologique hybride est développé et appliqué pour l'analyse numérique rhéologique d'un vieil éboulement situé dans Wanxian. Les résultats sont comparés aux phénomènes d'analyse et de macro-géologie de geomechanics, et montrent que la méthode présentée d'analyse est une approche efficace à l'analyse de mécanisme d'éboulement.

Key words: time-dependent deformation; rheological model; creep; visco-elasto-plastic FEM, landslide

INTRODUCTION

The rheology of rock is time-dependent deformation related characteristics under certain condition of stress and strain, including creep and relaxation. At present, based on the visco-elasto-plastic constitutive model, theoretical study and practical application of numerical analysis, the stability of surrounding rock and the time-dependent interacting analysis of rock-support in underground engineering, consolidation creep analysis of saturated soft clay and so on, had been obtained many meaningful achievements and experiences. In the analysis and study of slope deformation mechanism, because the slope stress and strain field gradually adjust with time, strength condition of rock also changes with time or even evolves to critical destabilizing condition. It is thus important to consider rheological effects in the analysis of propagation of deformation and failure mechanism of slope.

In terms of rock rheology, the reactivation of old landslides in interbedded sandstone and claystone, which are widely distributed on the Three Gorges side slopes, is analyzed based on the previous work of Huang (1983), Sun (1988), Keed Well (1984) and Christense (1984). A constitutive model is suggested and numerical analysis procedure of hybrid rheological model is presented in the paper. A visco-elasto-plastic FEM program is used in the analysis of time-dependent stress and strain of an old landslide located in the Three Gorges (Zhou 1995).

DEFORMATION MECHANISM OF THE REVIVAL OLD LANDSLIDE INTERBEDDED SANDSTONE AND CLAYSTONE

The broad distribution of Jurassic strata in the area of the Three Gorges dam consists of gentle terrain of claystone interbedded with sandstone, where unloading weathered joints are found everywhere. When surface water infiltrates into the joints, with the effect of polar water molecule, clay is activated and expands, and thus its shear strength reduces. During dehydration, claystone contracts under the overlying strata pressure. In the repeated course of expansion and contraction, anisotropic deformation develops in the claystone. With time and deformation developed, the directional arrangement of inner minerals causes the shear strength to deteriorate further. In the macroscopic aspect, claystone is apt to deform plastically toward the slope side. Interbedded sandstone deforms primarily in the form of visco-elastic deformation, while claystone in the form of visco-plastic deformation. Once the tensile stress along the bedding plane caused by the non-uniform deformation exceeds the tensile strength, sandstone would be split and disintegrated. Time-developed creeping extrusion in the claystone and the progressive crack evolution in the sandstone make the flat slope translate and disintegrate. In the coactions of dynamic, static water pressure field and gravitational field, the slope slips smoothly and forms the broad distribution of ancient landslide with interbedded sandstone and claystone in Three Gorges.

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HYBRID RHEOLOGICAL MODEL AND CONSTITUTIVE RULE

Considering the visco-elasto-plastic deformation feature of interbedded sandstone and claystone, the authors use a hybrid Kelvin-Voigt model and V.P. model (ideal visco-elasto-plastic model) in line as shown in Figure 1, to derive a creep relaxation constitutive equation by using Laplace transformation. The strain of this hybrid rheological model is:

$$\varepsilon = \varepsilon_e + \varepsilon_{ve} + \varepsilon_{vp} \tag{1}$$

For $\varepsilon_e = \sigma/E_1$, $\varepsilon_{ve} = \sigma/E_2 - \eta_1 \cdot \dot{\varepsilon}_{ve}/E_2$, $\dot{\varepsilon}_{vp} = (\sigma - f_s)/\eta_2$ and $\ddot{\varepsilon} = \ddot{\varepsilon}_e + \ddot{\varepsilon}_{ve} + \ddot{\varepsilon}_{vp}$, then we can obtain yield creep constitutive equation

$$\frac{\eta_1\eta_2}{E_2} \cdot \dot{\varepsilon} + \eta_2 \cdot \dot{\varepsilon} = \frac{\eta_1\eta_2}{E_1E_2} \cdot \ddot{\sigma} + (\frac{\eta_1}{E_2} + \frac{\eta_2}{E_2} + \frac{\eta_2}{E_1}) \cdot \dot{\sigma} + (\sigma - fs) \tag{2}$$

where $\eta_{1=}$ Voigt model coefficient of viscosity;
η_2 = V.P. model coefficient of viscosity;
f_s = yield damping force.

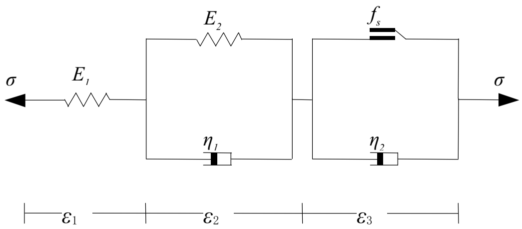

Figure 1. Hybrid rheological model

Discussion on the Yield Creep Constitutive Equation

When deriving Eq. (2), we make two assumptions: $\sigma > f_s$ and f_s=*constant.* However as Figure 2 shows: $t=0^-\sim0^+$, σ is a gradual increased variable, in a certain time σ reaches and then surpasses f_s; $t=0^-\sim0^+$ V.P. model plastic element yield stress f is also not a constant: $\sigma \le f_s$, f increases as σ increases; $\sigma > f_s$, $f=f_s$ (constant).

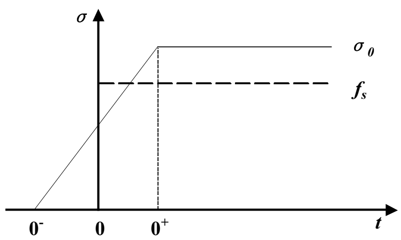

Figure 2. σ varies with time

So, in order to describe the variable character *of* σ and f, Heavyside unit step function is introduced

$$H(t) = \begin{cases} 1 & t > 0^+ \\ 0 & t < 0^- \end{cases} \tag{3}$$

where σ is given by

$$\sigma = \sigma_0 \cdot H(t) \tag{4}$$

f is given by

$$f = f_S \cdot H(t) \tag{5}$$

Taking f_s as variable of f, again by deducing Eq (2), the creep constitutive equation can be expressed as

$$b_2 \cdot \ddot{\varepsilon} + b_1 \cdot \dot{\varepsilon} + c_1 \cdot \dot{f} + f = \sigma + a_1 \cdot \dot{\sigma} + a_2 \cdot \ddot{\sigma} \tag{6}$$

where $b_2 = \eta_1 \cdot \eta_2 / E_2$,$b_1 = \eta_2$, $c_1 = \eta_1 / E_2$, $a_1 = (\eta_1 + \eta_2)/E_2 + \eta_2 / E_2$,$a_2 = \eta_1 \cdot \eta_2 / E_1 \cdot E_2$.

According to the definition of Laplace transform, $L[H(t)] = 1/s$, then the Laplace transform of σ, $\dot{\sigma}$, $\ddot{\sigma}$ is,

$$\left.\begin{array}{l} \hat{\sigma} = \sigma_0 / s \\ \hat{\dot{\sigma}} = \sigma_0 - \sigma(0) \\ \hat{\ddot{\sigma}} = s \cdot \sigma_0 - s \cdot \sigma(0) - \dot{\sigma}(0) \end{array}\right\} \tag{7}$$

We can also obtain the Laplace transform of f , $\dot{f}$, $\dot{\varepsilon}$ and $\ddot{\varepsilon}$ respectively.

When initial condition is t=0$^-$, make Laplace transform to Eq (2), then we can obtain ε Laplace transform $\hat{\varepsilon}$ as follows:

$$\hat{\varepsilon} = \frac{(\sigma_0 - f_S)}{b_2} \cdot \frac{1}{s^2 (s + \frac{b_1}{b_2})} + \frac{a_2 \cdot \sigma_0}{b_2} \cdot \frac{1}{(s + \frac{b_1}{b_2})} + \frac{(a_1 \sigma_0 - c_1 f_S)}{b_2} \cdot \frac{1}{s \cdot (s + \frac{b_1}{b_2})} \tag{8}$$

Make an inverse Laplace transform to Eq (8) and consider the meaning of a_1, a_2, b_1, b_2, c_1, then $\varepsilon(t)$ is derived as follows:

$$\begin{aligned} \varepsilon(t) = {} & \frac{(\sigma_0 - f_S)}{\eta_2} \cdot t - \frac{(\sigma_0 - f_S)\eta_1}{\eta_2 \cdot E_2} \cdot \left[1 - \exp\left(-\frac{E_2}{\eta_1} \cdot t \right) \right] + \\ & \frac{\sigma_0}{E_1} \exp\left(-\frac{E_2}{\eta_1} \cdot t \right) + \left(\frac{\eta_1 + \eta_2}{E_2} + \frac{\eta_2}{E_1} \right) \cdot \frac{\sigma_0}{\eta_2} \cdot \\ & \left[1 - \exp(-\frac{E_2}{\eta_1} \cdot t) \right] - \frac{\eta_1}{E_2 \eta_2} \cdot f_S \left[1 - \exp(\frac{E_2}{\eta_1} \cdot t) \right] \end{aligned} \tag{9}$$

Then

$$\varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{(\sigma_0 - f_S)}{\eta_2} \cdot t + \frac{\sigma_0}{E_2} \left[1 - \exp(-\frac{E_2}{\eta_1} \cdot t) \right] \tag{10}$$

When in loading, Eq (10) reflects instant elastic, delay elasticity and steady creep; while in unloading, it reflects delay restoring deformation and permanent deformation.

Building of Stress Relaxation Equation

when constant strain ε=ε_0, introducing step function $H(t)$ as:

$$\left.\begin{array}{l} \varepsilon = \varepsilon_0 \cdot H(t) \\ f = f_S \cdot H(t) \end{array}\right\} \tag{11}$$

When initial condition is set as t=0$^-$, marking Laplace transform of σ as $\hat{\sigma}$, then the Laplace transform to Eq (6) is:

$$\hat{\sigma} = \frac{1}{a_2 (s + \alpha)(s + \beta)} (\frac{f_S}{s} + c_1 f_S + b_1 \varepsilon_0 + b_2 \varepsilon_0 s) \tag{12}$$

where α 、$\beta = \frac{1}{2a_2}\left(a_1 \pm \sqrt{a_1^2 - 4a_2} \right)$.

By making inverse Laplace transform to Eq (12), relaxation equation can be obtained as follows:

$$\sigma(t) = \frac{f_s}{a_2 \alpha \beta} + K_1 \cdot \exp(-\alpha \cdot t) + K_2 \cdot \exp(-\beta \cdot t) \tag{13}$$

where

$$K_1 = \left[\left(\frac{1 - c_1 \cdot \alpha}{\alpha}\right) \cdot f_S + (b_2 \cdot \alpha - b_1) \cdot \varepsilon_0 \right] \Big/ a_2 \cdot (\alpha - \beta)$$

$$K_2 = \left[\left(\frac{1 - c_1 \cdot \beta}{\beta}\right) \cdot f_S + (b_2 \cdot \beta - b_1) \cdot \varepsilon_0 \right] \Big/ a_2 \cdot (\alpha - \beta)$$

VISCO-ELASTO-PLASTIC FEM APPLIED TO THE REVIVAL OLD LANDSLIDE INTERBEDDED SANDSTONE AND CLAYSTONE

The model in Figure 1 is taken as the rheological model. According to the above derivations, the deformation of the model consists of instant elastic deformation, visco-elastic deformation and visco plastic deformation.

Instant elastic deformation

According to Hooke's law

$$\left\{\Delta\varepsilon_e\right\} = \frac{1}{E_1}\left[A\right]\left\{\Delta\sigma\right\} \tag{14}$$

where E_1 =instant elastic modulus, [A]= constant matrix related to μ and was in the condition of plane strain.

Visco-elastic deformation

$$\left\{\dot{\varepsilon}_{ve}\right\}_n = \frac{\eta_1}{E_2}\left[A\right]\left\{\sigma\right\}_n - \eta_1\left\{\varepsilon_{ve}\right\}_n \tag{15}$$

In the period $\Delta t_n = t_{n+1} - t_n$, the increment of visco-elastic deformation is $\{\Delta\varepsilon_{ve}\}_n=\Delta t_n[(1-\Theta)\{\varepsilon_{ve}\}_n+\Theta\{\varepsilon_{ve}\}_{n+1}]$, using Euler time integral method in the condition of Θ=0, $\{\Delta\varepsilon_{ve}\}_n$ is derived as follows:

$$\left\{\Delta\varepsilon_{ve}\right\}_n = \frac{1}{E_2}\left[A\right]\left\{\sigma\right\}_n\left(1 - e^{-\frac{E_2}{\eta_1}\cdot\Delta t_n}\right) - \left(1 - e^{-\frac{E_2}{\eta_1}\cdot\Delta t_n}\right)\cdot\left\{\varepsilon_{ve}\right\} \tag{16}$$

where E_2 = delay elastic modulus, η_1 = visco-elastic coefficient

Visco-plastic deformation

The material yield is checked with visco-plastic yield criterion,. If the material is in plastic yield state, then the deformation of this part should be considered. Druck-Prager criterion is used to represent visco plastic yield.

$$F = \frac{6\cdot\sin\varphi}{3-\sin\varphi}\cdot\sigma_m + \sqrt{3J_2} - \frac{6\cdot c\cdot\cos\varphi}{3-\sin\varphi} \tag{17}$$

where $\sigma_m = I_1/3 = \sigma_{ij}/3$, $J_2=S_{ij}\cdot S_{ij}/2$.

In time period $\{\Delta t_n\}$, visco-plastic strain and plastic strain have the similar variation, and visco-plastic flow criterion in complex stress state is expressed as :

$$\left\{\dot{\varepsilon}_{vp}\right\} = \frac{1}{\eta_2}\left\langle\phi(F)\right\rangle\left\{\frac{\partial G}{\partial\sigma}\right\} \tag{18}$$

where η_2 = visco plastic flow coefficient, G= plastic potential function, $<\Phi(F)>$= switch function given by:

$$\left\langle\phi(F)\right\rangle = \begin{cases} \phi(F) = e^{\left(\frac{F-F_0}{F_0}\right)} & F > 0 \\ 0 & F \le 0 \end{cases} \tag{19}$$

In this calculation, associate flow criterion is used as $G \equiv F$.

In time period of $\Delta t_n=\Delta t_{n+1} - t_n$, the increment of visco-plastic strain is calculated as :

$$\left\{\Delta\varepsilon_{vp}\right\}_n = \Delta t_n\left[\left(1-\Theta\right)\left\{\dot{\varepsilon}_{vp}\right\}_n + \Theta\left\{\dot{\varepsilon}_{vp}\right\}_{n+1}\right] \tag{20}$$

Euler explicit time integral method is used in the condition of Θ=0, according to previous deduction, the increment of visco deformation is $\{\Delta\varepsilon_{ve}\}_n=\Delta t_n[(1-\Theta)\{\varepsilon_{ve}\}_n+\Theta\{\varepsilon_{ve}\}_{n+1}]$ in Δt_n, then the visco loading correction term is:

$$\{F_v\}_n = \int_\Omega [B]^T \cdot [D] \cdot \{\Delta\varepsilon_v\}_n \, d\Omega \tag{21}$$

Using general equilibrium equation $[K]\{\delta(t)\} = \{F\} + \{F_v\}_n$ compute visco strain at next time-interval, and then this visco strain is used as the initial strain of next iteration, repeat this procedure until convergence and stability condition is satisfied.

CASE STUDY

As a result of the construction of the Three Gorge Project, most of Wanxian city will be inundated and part of the new city will be moved to an area of old landslides and nearby locations. Based on the analysis of a qualitative developing mechanism, the old landslides in this area basically belongs to the integral slippage bedrock landslide type of flat interbedded sandstone and claystone. Field monitoring shows that creep of the rock is still developing, the rock located on the two sides and in front of the old landslides, especially in rainy period or following them, some rock deformation becomes even worse. For controling and using of the old landslides, it is necessary to predict stress and strain variation in the old landslides and the trend of development with time.

Two conditions are considered, namely natural stress and strain field; and natural stress state combined with dynamic water pressure field caused by heavy storm rainfall. A rheological FEM is used in analyses of some old landslides. Localised and overall time-dependent stress and strain fields for the old landslide are discussed in the paper. The analysis results are coincident with the deformation characteristics and phenomena of the present old landslides.

Finite element model

Figure 3 shows the finite element model and grid. Table 1 lists the parameters of rock mass in the model.

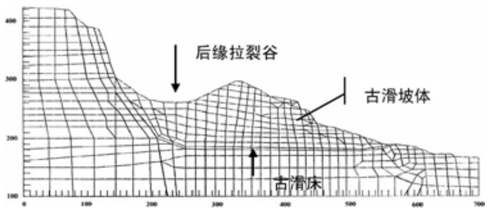

Figure 3. Finite element model and grid

Table 1 Rock medium calculated parameter

Rock	E (MPa)	μ	η_{ve} (η_l) (poise)	η_{vp} (η_l) (poise)
Sandstone	15000	0.24	3.0×10^{12}	8.0×10^{11}
Claystone	5534	0.29	3.0×10^{11}	8.0×10^{10}

Result analysis

Figure 4a and 4b show the isolines of σ3 (contours of the minor principal stress) at the end of 1Δt and the end of 45Δt. After considering the old landslides natural structural character, the inducting rheological effect and the effect of the underground seepage field, bedrock clay argillation creep, and plastic flow occurs toward the free surface of the slope. The tension stress zone develops at the contact surface and posterior margin of sandstone, claystone, which leads to rock strength deterioration.

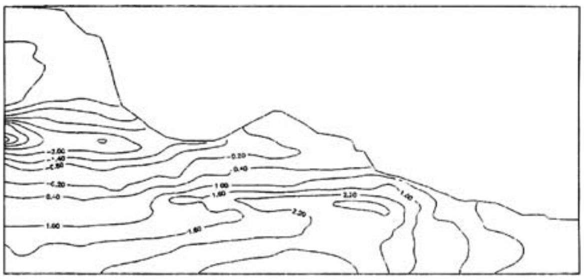

Figure 4a. Isoline of σ3 of the end of 1Δt

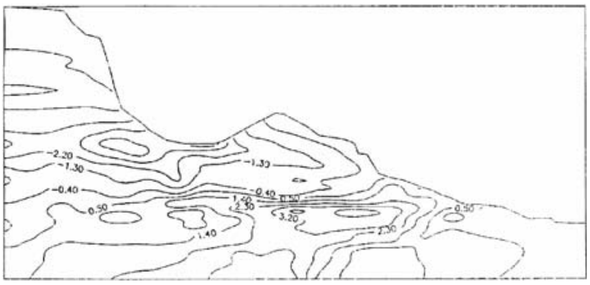

Figure 4b. Isoline of σ3 of the end of 45Δt

The deformation zone is mainly located in the old landslide area. Figure 5a and 5b shows the isolines of horizontal displacement at the end of 1Δt and the end of 45Δt. The horizontal displacement at the end of 1Δt is not clear, only clear in front of slope with a maximum of 2 mm. At the end of 45Δt, in front of the slope, the maximum horizontal displacement UX reaches 91.8 mm. The isoline of UY indicates that UY of scarp crack developed from 0.5 mm initially to over 40 mm. Displacement vector indicates that at first displacement vector is not clear, but at the end of 45Δt it increase notablely toward the slope of the old landslide, and local settlement at the scarp of the old landslide occurs and the integral flat slide tendency along the bedding surface develops.

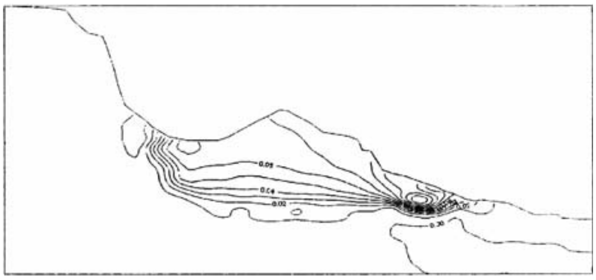

Figure 5a. Isoline of horizontal displacement of the end of 1**Δt**

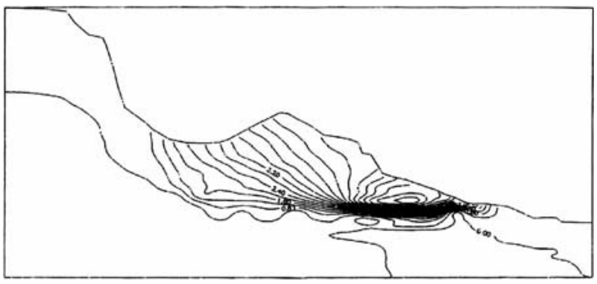

Figure 5b. Isoline of horizontal displacement of the end of 45Δt

Figure 6 shows how the process of rock failure region gradually develops with time. Since the old landslide bed is a relative weak interlayer, the second deformation of the slope develops along old landslide bed. After developing to the limiting state, a new potential slip plane inheriting the old landslide bed is formed. At the same time, because the horizontal creep increases the tensile deformation of the scarp, sliding along bedrock causes the rock near the scarp to fall and roll down the slope, which is another geohazard.

According to the site monitoring result of ground displacement in this area, at present the whole old landslide is steady. However, the front of the slope and part of scarp shows sign of remarkable deformation. Especially in the period of continuous rainfall, the secondary creeping deformation of the old landslide is particularly obvious, and new tensile cracks appear on the top of the scarp. Sometimes, there are some geological catastrophes such as rock collapse and toppling, which can unexpectedly affect agriculture productivity and inhabitation. The real situation is basically consistent with the calculated result in the paper.

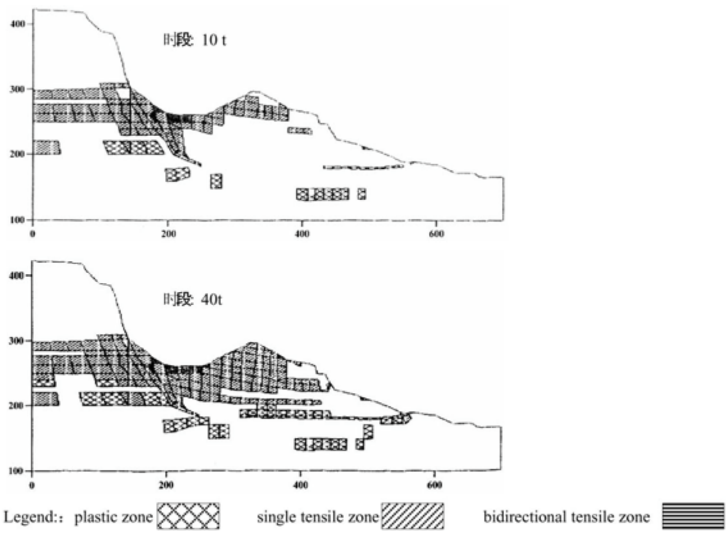

Figure 6. Progressive failure of slope

CONCLUSIONS

Based on the understanding of long-term time dependent deformation of the old landslide, the creep deformation and the failure of the ancient landslide, which was in interbedded sandstone and claystone, is discussed. A hybrid model describing the elasto-viscous, elasto-plastic deformation characteristics of complex landslides is derived. The deduction of the creeping, relaxation constitutive equations is made by means of Laplace transformation. Creeping and relaxation of the model is discussed.

Based on the hybrid rheological model, a finite element model consisting of the sandstone, claystone and the visco-elastic, visco-plastic strain of the weak interlayer was set up. The old landslide in the inundated area of Wanxian City in the Three Gorge reservoir is analyzed by the rheological finite element.

Considered the extreme situation with the seepage field caused by intense rainfall, the result of calculation shows that the old landslide will keep its overall translation of secondary creeping deformation of the original old landslide. As a result of rock creeping, the long-term strength of rock reduces and collapse and topple of scarp rocks occurs.

When using the old landslide as a site for city exploitation and construction, the cause of formation, composition, features of deformation development of the old landslide (including the necessary numerical analysis) should be investigated sufficiently, by the principle of protection, adaptability, using comprehensive management such as establishing a deformation monitoring system and necessary artificial drainage system, strengthening the landslide using cable and shotcrete.

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