Bayesian updating for improving the accuracy and precision of pile capacity predictions

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Abstract: To estimate and obtain the correct capacities of piles bored in the field, plenty of different formulas have been developed to solve this problem. Theoretical equations and empirical or semi-empirical methods have been applied to obtain the pile capacity including the static and dynamic methods. Developing and obtaining these predictions are particularly complex.

Overcoming the complications of predicting pile capacity is mainly due to the correlation methods that are applied to standard penetration tests (SPT), cone penetration tests (CPT), and pressure meter tests. Although these tests reflect to some extent the natural soil conditions, they also include many limitations. However, these tests may provide good predictions if correlated with load test data on a regional basis.

Because of the limitations from correlations obtained with the in situ tests, many empirical formulas have been developed based on load test results to provide quick and easy methods to estimate pile capacity. The most widely used empirical methods are proposed by Meyerhof (1976), Coyle and Castello (1981), Briaud et al. (1989), the American Petroleum Institute (RP2A, 1993), SPT97 (1995). Use of these methods has shown that they either oversimplify or improperly consider the effects of residual stresses, actual soil parameters, and the stress history. Therefore, it is necessary to develop an alternative method using the appropriate inputs to predict the pile capacity.

The design of pile foundations and the estimation of static pile capacities based on measured soil properties have improved considerably over the years. Neural network paradigms are used to develop suitable computational models for calculating pile foundations, and it uses only simple geotechnical soil parameters and pile properties as the necessary inputs. In this study, the model of Artificial Neural Network used for predicting pile capacity, includes Back-Propagation Neural Networks (BPNN) and Generalized Regression Neural Networks (GRNN).

Using the prior and likelihood distribution to deal with the question of updating the pile capacity. Bayesian technique is applied to updates of the predictions of axial pile capacity. The updated posterior probability values present the best estimation of pile capacity.

Résumé: Estimer et obtenir les capacités correctes de piles étaient ennuyeux dans le domaine, abondance de différentes formules ont été développés pour résoudre ce problème. Des équations théoriques et les méthodes empiriques ou de semi-finale-empirical ont été appliquées pour obtenir la capacité de pile comprenant les méthodes statiques et dynamiques. Développer et obtenir ces prévisions sont particulièrement complexes.

Surmonter les complications de la capacité de prévision de pile est principalement dû aux méthodes de corrélation qui sont appliquées aux essais de pénétration standard (SPT), aux essais de pénétration de cône (CPT), et aux essais de mètre de pression. Bien que ces essais reflètent dans une certaine mesure les conditions normaux de sol, ils incluent également beaucoup de limitations. Cependant, ces essais peuvent fournir de bonnes prévisions si corrélé avec des essais de charge sur une base régionale.

En raison des limitations des corrélations obtenues avec les essais in situ, beaucoup de formules empiriques ont été développées ont basé sur des résultats d'essai de charge pour fournir des méthodes rapides et faciles pour estimer la capacité de pile. Meyerhof (1976), Coyle et Castello (1981), Briaud et autres (1989), l'institut américain de pétrole (RP2A, 1993), SPT97 (1995) proposent les méthodes empiriques le plus largement répandues. L'utilisation de ces méthodes a prouvé qu'ils trop simplifient ou considèrent incorrectement les effets des efforts résiduels, des paramètres réels de sol, et de l'histoire d'effort. Par conséquent, il est nécessaire de développer une méthode alternative en utilisant les entrées appropriées pour prévoir la capacité de pile.

La conception des bases de pile et l'évaluation des capacités statiques de pile basées sur les propriétés mesurées de sol se sont améliorées considérablement au cours des années. Des paradigmes de réseau neurologique sont employés pour développer les modèles informatiques appropriés pour les bases calculatrices de pile, et il emploie seulement des paramètres de sol et des propriétés géotechniques simples de pile comme entrées nécessaires. Dans cette étude, le modèle du réseau neurologique artificiel utilisé pour prévoir la capacité de pile, inclut les réseaux neurologiques d'En arrière-Propagation (BPNN) et les réseaux neurologiques généralisés de régression (GRNN).

En utilisant la distribution antérieure et de probabilité pour traiter la question de mettre à jour la capacité de pile. La technique bayésienne est appliquée aux mises à jour les prévisions de la capacité axiale de pile. Les valeurs postérieures mises à jour de probabilité présentent la meilleure évaluation de la capacité de pile.

Keywords: Case studies, contaminated land, groundwater provinces, regional planning, underground installations.

INTRODUCTION

To estimate pile capacity, methods such as the SPT97, American Petroleum Institute (API), and Generalized Regression Neural Networks (GRNN), to name just a few (Meyerhof, 1976; Coyle & Castello, 1981; Briaud, Tucker

& Eng, 1989; the American Petroleum Institute (API) RP2A, 1993; SPT97, 1995). Comparisons between measured and predicted capacities for each method show large scatter. These empirical methods yield only approximate predictions. It should be noted that the pile load test itself involves some degree of uncertainty and does not necessarily represent the actual ultimate capacity. Svinkin, Morgano & Morvant (1994) reported that pile capacity changes with time, as the bearing capacity of the pile is not fully mobilized at the time of initial driving of the pile. In addition, the load test may not be carried out to "failure" and the ultimate capacity has to be "interpreted" from the results of the load test. Nevertheless, pile load testing represents the most accurate way to determine the ultimate capacity of a pile.

Use of load test results to calibrate the prediction from the static analysis method and to reduce the uncertainty of the prediction, is often employed. In fact, the generally accepted practice in pile design (e.g., see National Highway Institute's driven pile manual compiled by Hannigan et al., 1998) calls for use of a factor of safety (FS) of 2 when pile load test is to be performed, and a higher FS needed to be used (e.g. could be as high as 3.5 or 4) when the load test is not prescribed. Zhang (2004) showed that the reliability of a pile design using the traditional static analysis method could be enhanced by the new information obtained from pile load tests. The Bayesian updating technique has been shown to be effective in updating the reliability given the pile load test results (Lacasse, Guttormsen & Goulois, 1990).

Use of the Bayes' theorem for updating pile capacity is of course not new. Kay (1976) proposed the Bayesian statistical approach to combine prior information of estimated pile capacity with load test results for single piles in sand. Sidi and Tang (1987) considered model error associated with the prediction of pile capacity in clay based on load test results and pile driving data. Lacasse, Tan & Keaveny (1991) used the Bayesian technique to update the prediction of pile driving and axial pile capacity on the basis of pile driving records. Sugai & Matsuo (1993) proposed a method to update the pile set-up factor for piles in clay. Zhang & Tang (2002) used the Bayesian approach to incorporate the results of pile load tests into pile design (e.g., for reducing pile length on the basis of the same target reliability index).

In this paper, the Bayesian updating technique is used systematically to improve the accuracy of pile capacity prediction. To apply the Bayesian updating technique in this study, the values of bias factor (the ratio of measured capacity over predicted capacity) are treated as prior distribution, and previous experience and results obtained by others (e.g., Zhang & Tang, 2002) regarding pile load tests are treated as the likelihood distribution. Three types of soils considered are sand, clay, and mixed soils.

BAYESIAN FRAMEWORK – PARAMETER ESTIMATION AND BAYESIAN STATISTICS

Bayesian updating is a well-established technique. The reader is referred to the literature of classical reliability theory for details (Ang & Tang, 1975; Ayyub & McCuem, 1997). A brief summary of the Bayesian framework is presented here as a working background.

Discrete Parameters

For an unknown parameter Θ, a prior distribution for the parameter can be subjectively determined and expressed as:

$$P_{\Theta}(\theta_i) = P(\theta_i) \quad \text{for } i = 1, 2, ..., n \tag{1}$$

The parameter Θ is assumed to take *n* discrete values with probabilities and the distribution of Θ reflects the uncertainty in the parameter including its randomness. Let ε denote the observed outcome of the experiment. If event A occurred, what is the probability that a particular event E_i also occurs? The desired probability can be obtained by applying Bayes' theorem, we have:

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} \qquad i = 1, 2, \dots, n \tag{2}$$

Then, the posterior (updating) distribution of the parameter can be obtained as:

$$P(\Theta = \theta_i|\varepsilon) = \frac{P(\varepsilon|\Theta = \theta_i)P(\Theta = \theta_i)}{\sum_{i=1}^{n} P(\varepsilon|\Theta = \theta_i)P(\Theta = \theta_i)} \qquad i = 1, 2, \dots, n \tag{3}$$

where $P(\varepsilon|\Theta = \theta_i)$: the likelihood of the experimental outcome ε if $\Theta = \theta_i$, $P(\Theta = \theta_i)$: the prior probability of $\Theta = \theta_i$, and $P(\Theta = \theta_i|\varepsilon)$: the posterior probability of $\Theta = \theta_i$.

Denoting the prior and posterior probabilities as $P'(\Theta = \theta_i)$ and $P''(\Theta = \theta_i)$, respectively, Equation (3) becomes as:

$$P''(\Theta = \theta_i) = \frac{P(\varepsilon|\Theta = \theta_i)P'(\Theta = \theta_i)}{\sum_{i=1}^{n} P(\varepsilon|\Theta = \theta_i)P'(\Theta = \theta_i)} \tag{4}$$

Continuous Parameters

For an unknown random variable, Θ, with a prior density function $f'(\theta)$ shown in Figure 1, the prior probability that θ will fall between θ_i and $\theta_i + \Delta\theta$ is $f'(\theta_i)\Delta\theta$.

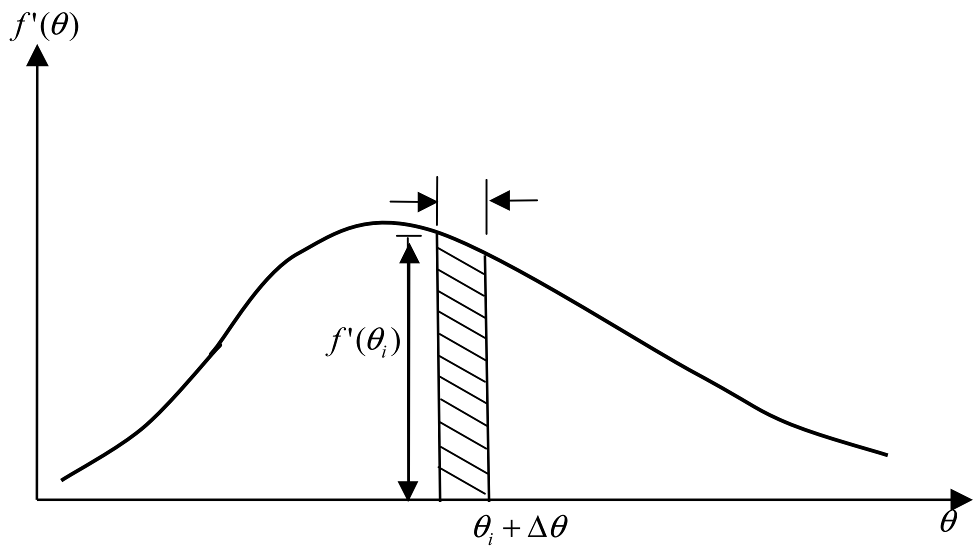


Figure 1. Continuous prior distribution of parameter θ (Ang and Tang, 1975)

If ε is an observed experimental outcome, the posterior probability that θ will fall between θ_i and $\theta_i + \Delta\theta$ is:

$$f''(\theta_i)\Delta\theta = \frac{P(\varepsilon|\theta_i) f'(\theta_i)\Delta\theta}{\sum_{i=1}^{n} P(\varepsilon|\theta_i) f'(\theta_i)\Delta\theta} \tag{5}$$

where $P(\varepsilon|\theta_i) = P(\varepsilon|\theta_i < \theta \le \theta_i + \Delta\theta)$.

As $\Delta\theta$ approaches zero, Equation (5) becomes:

$$f''(\theta_i) = \frac{P(\varepsilon|\theta_i) f'(\theta_i)}{\int_{-\infty}^{\infty} P(\varepsilon|\theta) f'(\theta) d\theta} \tag{6}$$

The term $P(\varepsilon|\theta)$ is the condition probability and is commonly referred to as the likelihood function of θ and denoted as $L(\theta)$. It follows that Equation (6) can be expressed as:

$$f''(\theta) = K\, L(\theta) f'(\theta) \tag{7}$$

where K = the normalizing constant, $K = [\int_{-\infty}^{\infty} L(\theta) f'(\theta) d\theta]^{-1}$, $L(\theta)$ = the likelihood function, $f'(\theta)$ = the prior distribution, and $f''(\theta)$ = the posterior density function.

Bayesian Statistics

Bayesian updating uses the available pile load test data from each category. Here, the bias factor is treated as the prior distribution. Table 1 lists the values of typical "within-site" coefficient of variation (COV) of the driven pile capacity from load tests taken from sites, compiled by Zhang & Tang (2002). Based on these results, Zhang & Tang

(2002) suggested that the likelihood distribution function might be taken as a lognormal distribution with a mean of 1.0 and a COV of 0.20.

Table 1. Within-Site variability of the capacity of driven piles (Zhang & Tang, 2002)

Sites	Numbers of Pile	Soil Conditions	COV
Ashdod, Israel	12	Sand	0.22
Bremerhaven,Germany	9	Sand	0.28
San Franciso, U.S.A.	5	Sand and Clay	0.27
Southern, Italy	12	Sand and Gravel	0.25
Southern, Italy	4	Sand and Gravel	0.12
Southern, Italy	17	Sand and Gravel	0.19
Southern, Italy	3	Sand and Gravel	0.12
Southern, Italy	4	Sand and Gravel	0.14
Southern, Italy	16	Clay, Sand and Gravel	0.20

In the reliability analysis presented here, the pile capacity (resistance) and loads are considered to be log-normally distributed. It is necessary to calculate the first two moment parameters, the mean and standard deviation, of a log-normal distribution from that of the corresponding normal distribution. It can be shown that (Haldar & Mahadevan, 2000):

$$\lambda_X = E(\ln x) = \ln \mu_X - \frac{1}{2}\zeta_X^2 \tag{8}$$

and

$$\zeta_X^2 = \ln\left[1+\left(\frac{\sigma_X}{\mu_X}\right)^2\right] \tag{9}$$

where λ_X: the lognormal mean value, ζ_X: the lognormal standard deviation, μ_X: the normal mean value, and σ_X: the normal standard deviation,

Based on the converted normal distributions of the prior information and the likelihood information, Bayesian updating yields the mean and standard deviation of the updated (posterior) distributions as follows (Ang & Tang, 1975):

$$\mu'' = \frac{\mu'\sigma^2 + \mu\sigma'^2}{\sigma'^2 + \sigma^2} \tag{10}$$

and

$$\sigma''^2 = \frac{\sigma^2\sigma'^2}{\sigma'^2 + \sigma^2} \tag{11}$$

where μ' : the mean value of prior distribution, μ : the mean value of likelihood distribution, μ'' : the mean value of updated (posterior) distribution, σ' : the standard deviation of prior distribution, σ : the standard deviation of likelihood distribution, σ'' : the standard deviation of updated (posterior) distribution.

The updated statistics of the mean and standard deviation are then converted back to normal distribution as follows (Zhang & Tang, 2002):

$$\sigma_p'' = \mu_p''\sqrt{\exp(\sigma''^2) - 1} \tag{12}$$

and

$$\mu_p'' = \exp(\mu'' + \frac{1}{2}\sigma''^2) \tag{13}$$

where σ_p'' : the updated standard deviation of the converted normal distribution, σ'' : the updated standard deviation of the lognormal distribution, μ_p'' : the updated mean value of the converted normal distribution, and μ'' : the updated mean value of lognormal distribution.

DATABASE

A large database of pile load tests collected by McVay (2003, personal communication) is used in this study. This database includes 125 examples; 75 are concrete piles (both square and circular), 14 are steel pipe piles, and 36 are H-piles. For each pile in the database, the detailed geotechnical profile, in-situ tests data and the pile load test results are included. The data includes site location, pile types and geometry, pile length, pile properties, pile load test data, pile driving information, and the information of soil stratigraphy (including the ground surface elevation, water table elevation, soil description, and soil type) and the SPT N-value. The failure load listed was obtained from the load-settlement curve using Davisson's criterion.

BAYESIAN UPDATING - RESULTS AND DISCUSSIONS

For each pile capacity prediction method, the mean and COV of the bias factor determined are the prior information. The prior distribution is shown in Figure 2 along with the likelihood distribution and the posterior (updated) distribution in conjunction with the use of each pile capacity prediction method.

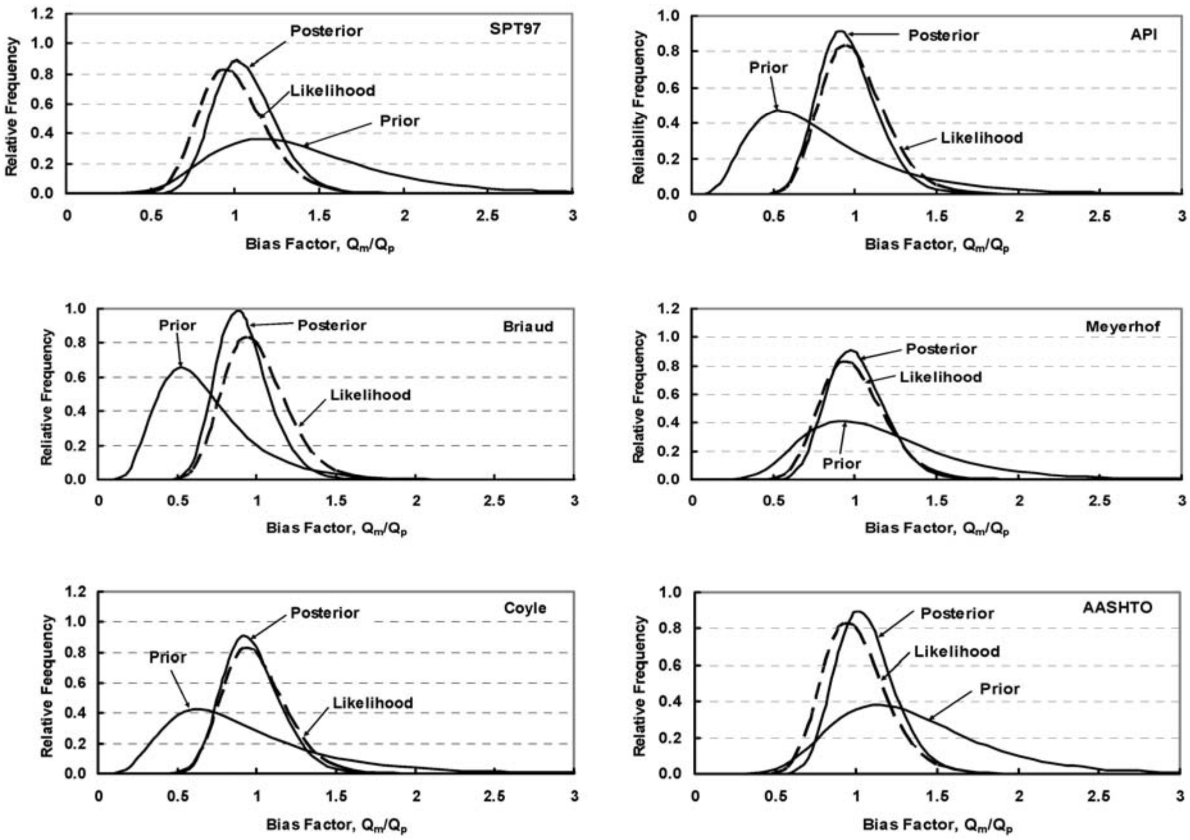


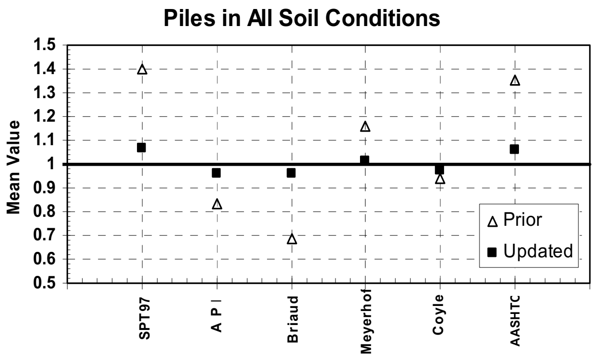
Figure 2. Prior, likelihood, and posterior distribution function for all piles in all soil condition

In each Bayesian updating, the likelihood distribution from pile load tests is set to be a lognormal distribution with a mean of 1.0 and a COV of 0.20, as discussed previously. Using the Bayesian statistics formulation summarized previously, the mean and COV of the posterior (updated) distribution are obtained. Table 2 presents the results of Bayesian updating for pile capacity prediction methods using all piles in the database.

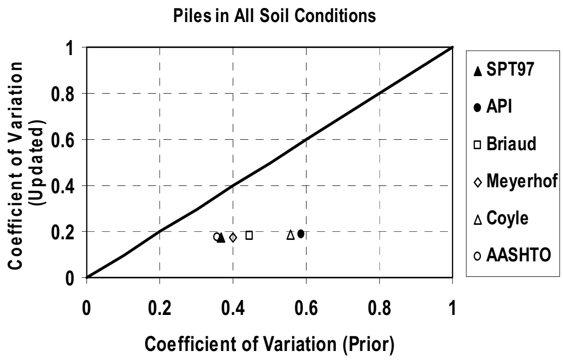
Table 2. The Bayesian updated results for prediction methods in all data

All Data	SPT 97		API		Briaud		Meyerhof		Coyle		AASHTO	
	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
Prior	1.399	0.367	0.836	0.588	0.686	0.447	1.163	0.400	0.942	0.558	1.356	0.358
Updated	1.067	0.174	0.962	0.188	0.960	0.181	1.016	0.177	0.976	0.187	1.062	0.173

As noted previously, some prediction methods (such as the API and the Coyle methods) are much less *precise*, as evidenced by their much higher COV values in the calculated bias factor. However, after updating with the likelihood function (from within-site pile load tests), the COV for all prediction methods are greatly reduced and the mean values all become much closer to 1.0. Figures 3(a) and 3(b) show plots of the means and COVs before and after updating. The significance of pile load tests on the improved accuracy of pile capacity prediction is clearly demonstrated.



(a)



(b)

Figure 3. (a) The comparison of mean values of all the data before and after updating; (b) The comparison of coefficient of variations of all the data before and after updating

It is also interesting to note that greater improvement occurred in the prediction methods that performed poorly before updating, and that regardless of which pile prediction method is initially adopted, the COV of the bias factor after updating is strongly influenced by the COV of the likelihood function, and the mean of the bias factor after updating is approximately equal to 1.0. This implies that the within-site pile load test can be an effective tool to calibrate any of the prediction methods examined, which is an approach that is often adopted by the engineer for pile design in a large project where a large number of piles are to be used. In the results presented previously, the Bayesian updating analysis was carried out using the prior distribution determined by using all piles in the database. Tables 3, 4, and 5 list the prior information based on piles only in clay sites, sand sites, and mixed soil sites, respectively, and their corresponding updated distributions.

Table 3. The Bayesian updated results for prediction methods in clay sites

Clay Site	SPT 97		API		Briaud		Meyerhof		Coyle		AASHTO	
	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
Prior	1.322	0.279	1.034	0.396	0.894	0.558	1.126	0.413	0.911	0.464	1.142	0.209
Updated	1.087	0.161	0.992	0.177	0.969	0.187	1.008	0.178	0.968	0.182	1.055	0.144

Table 4. The Bayesian updated results for prediction methods in sand sites

Sand Site	SPT 97		API		Briaud		Meyerhof		Coyle		AASHTO	
	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
Prior	1.593	0.410	0.863	0.579	0.742	0.435	1.321	0.492	1.125	0.537	1.423	0.333
Updated	1.081	0.178	0.966	0.187	0.931	0.180	1.026	0.184	0.999	0.186	1.085	0.170

Table 5. The Bayesian updated results for prediction methods in mixed soil sites

Mixed Soil Site	SPT 97		API		Briaud		Meyerhof		Coyle		AASHTO	
	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
Prior	1.405	0.425	0.805	0.705	0.600	0.380	1.205	0.506	0.837	0.658	1.411	0.484
Updated	1.050	0.179	0.964	0.191	0.878	0.176	1.010	0.184	0.965	0.190	1.038	0.183

Figures 4, 5, and 6 show the plots of prior, likelihood, and posterior distributions for piles in clay sites, sand sites, and mixed soil sites, respectively.

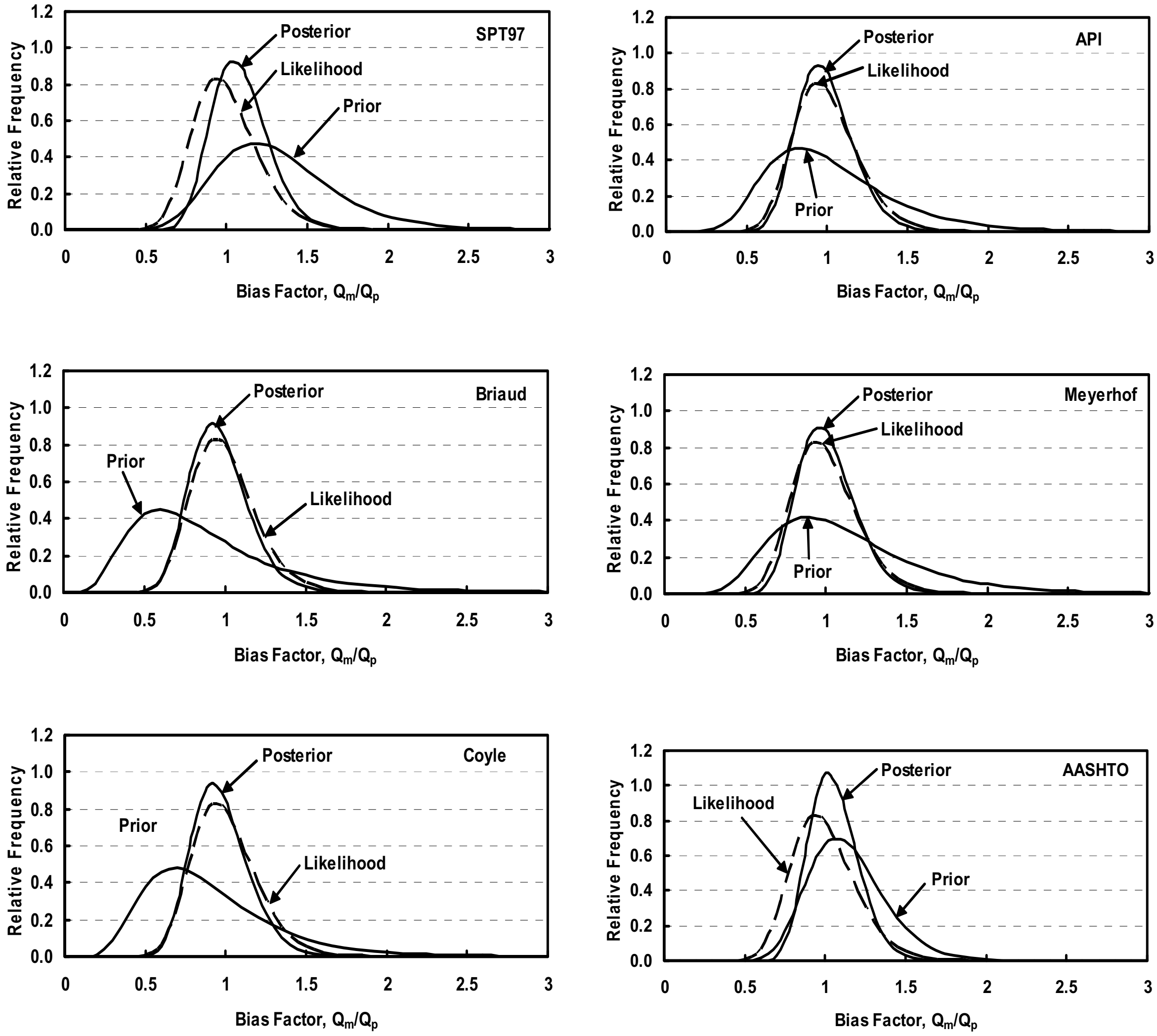


Figure 4. Prior, likelihood, and posterior distribution function for piles in clay sites

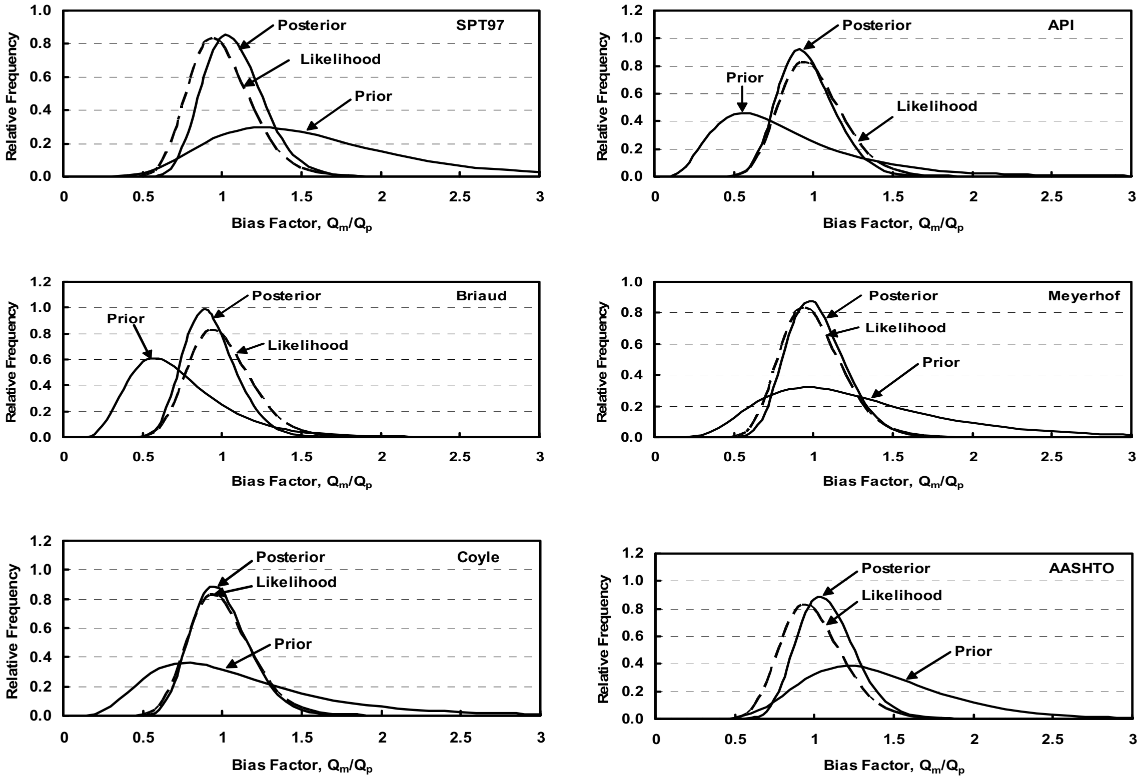


Figure 5. Prior, likelihood, and posterior distribution function for piles in sand sites

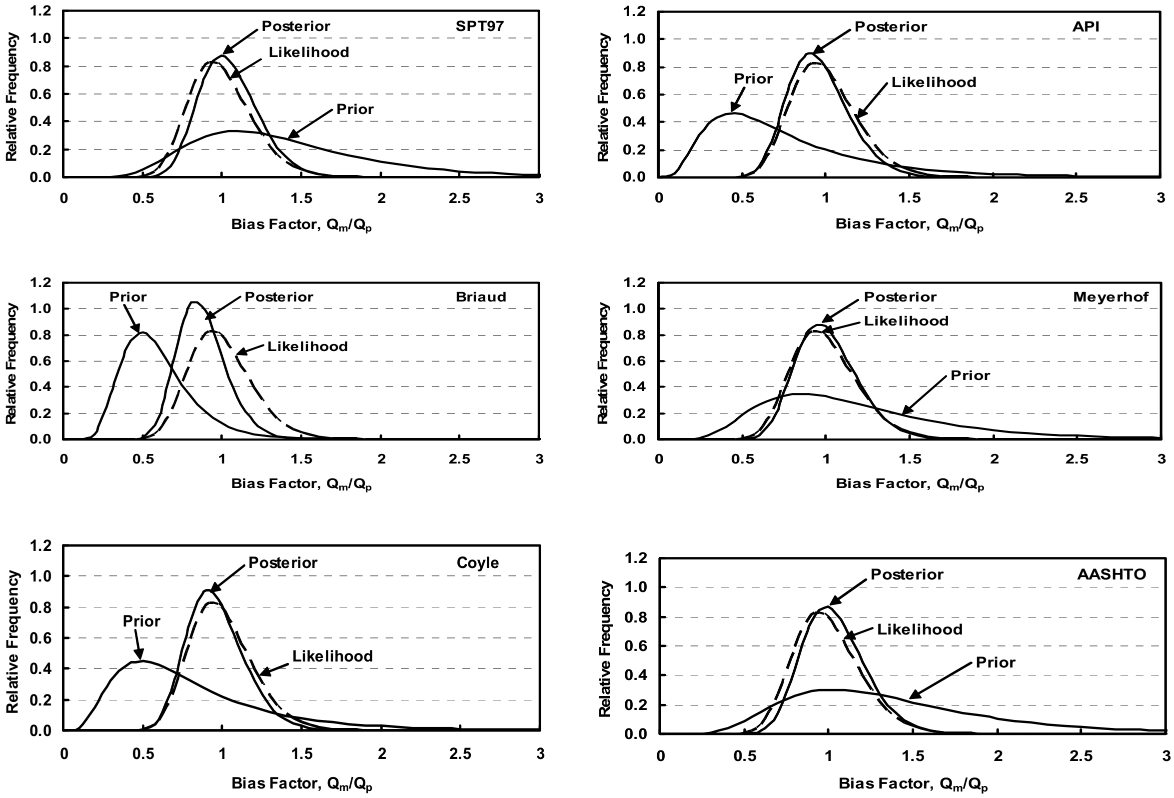


Figure 6. Prior, likelihood, and posterior distribution function for piles in mixed soil sites

Similar observations for piles in each of the three soil conditions, as those presented previously for piles in all soil sites, can be made. This is evident from the comparison of the mean value of the bias factor by different pile prediction methods for piles in different sites before and after updating, shown in Figure 7, and the comparison of the COV of the bias factor before and after updating shown in Figure 8.

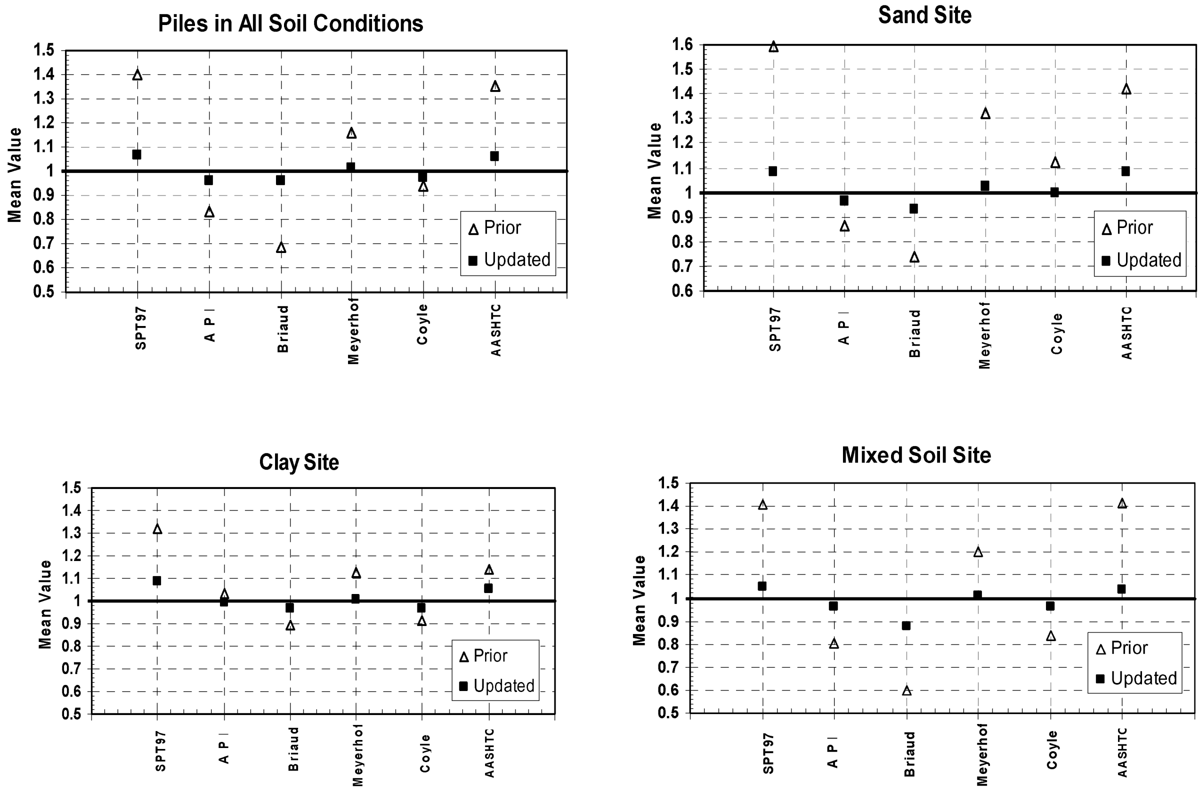


Figure 7. The comparison results of mean value in different soil sites

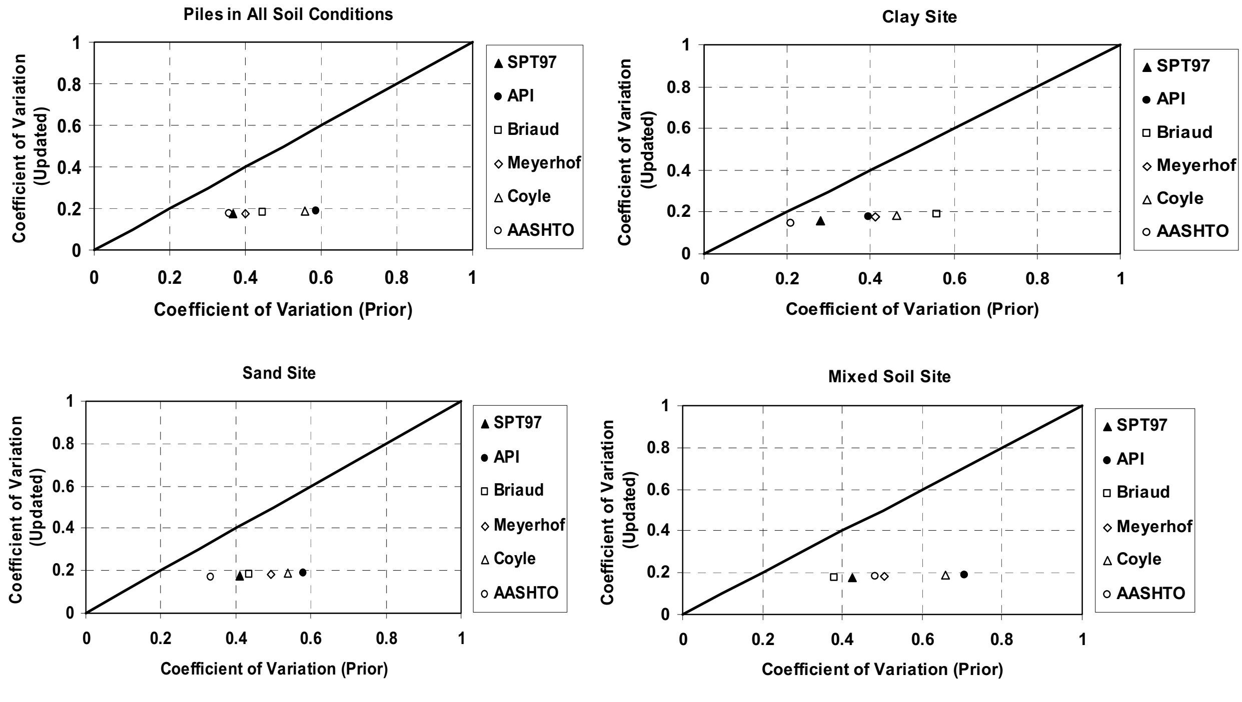


Figure 8. The comparison results of the coefficient of variation in different soil site.

It should be noted that in an actual application, pile capacity is calculated by a particular static analysis method, say, the AASHTO method. By applying the prior distribution of the bias factor established for the AASHTO method in this study, the distribution of the pile capacity by the AASHTO method for this particular pile at this particular site is obtained. If pile load tests are performed for similar piles at this site, the likelihood function can be established, which should be similar to the one assumed in this paper (the mean of the bias factor equal to 1.0 and the COV of the bias factor equation to 0.20). By applying the Bayesian updating, the posterior distribution of the pile capacity of the pile examined can be obtained. The posterior information provides an improved estimation of the pile capacity.

Moreover, the posterior distribution of the pile capacity (or the bias factor) can be used in lieu of the prior distribution in the calculation of reliability index. The updated reliability index using the updated distribution of bias factor provides a simple means for updating resistance factor for the same target reliability index. This resistance factor updating is very important and beneficial because if the resistance factor can be "raised" at the same target reliability index due to additional information derived from pile load tests, then the pile capacity is "allowed" to increase for the same pile subjected to the same soil conditions or alternatively, the pile length (or the number of piles in a pile group) can be reduced to maintain the same pile capacity and thus reduce the project cost.

CONCLUSIONS

The Bayesian updating technique is shown to be effective in reducing the uncertainty in pile capacity predicted by the prediction methods to various extents. For the methods that have a higher COV in the bias factor prior to updating, the updating with the likelihood function based on typical pile load tests drastically reduces the COV of the bias factor; for the methods that have a much lower COV in the bias factor prior to updating, some improvement in the accuracy and precision of the prediction is achieved, albeit not as significant or drastic. Regardless of which pile prediction method is initially adopted, the COV of the bias factor after updating is strongly influenced by the COV of the likelihood function, and the mean of the bias factor after updating is approximately equal to 1.0. This implies that the within-site pile load test can be an effective tool to calibrate any of the prediction methods examined.

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