Analysis of permeability characteristics along rough-walled fractures

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Abstract: This study is conducted to calculate the permeability coefficient in a single fracture while taking the true fracture geometry into consideration. The fracture geometry is measured using the confocal laser scanning microscope (CLSM). The CLSM geometry data are used to reconstruct a fracture model for numerical analysis using a homogenization analysis (HA) method. HA is a new type of perturbation theory developed to characterize the behavior of a micro-inhomogeneous material that involves periodic microstructures. HA permeability is calculated based on the local geometry and material properties (water viscosity in this case). The results show that the permeability coefficients do not follow the theoretical relationship of the cubic law.

Résumé: Cette étude est entreprise pour calculer le coefficient de perméabilité dans une rupture simple tout en prenant en compte la véritable géométrie de rupture. La géométrie de rupture est mesurée à l'aide du microscope confocal de balayage de laser (CLSM). Les données de la géométrie de CLSM sont employées pour reconstruire un modèle de rupture pour l'analyse numérique en utilisant une méthode de l'analyse d'homogénéisation (AH). AH est un nouveau type de théorie de perturbation développé pour caractériser le comportement d'un matériel micro-non homogène qui implique des microstructures périodiques. L'AH de perméabilité est calculé a basé sur la géométrie locale et les propriétés matérielles (viscosité de l'eau dans ce cas-ci). Les résultats prouvent que les coefficients de perméabilité ne suivent pas le rapport théorique de la loi cubique.

Keywords: permeability, fractures, joints, rock mechanics, hydrogeology

THEORY OF HOMOGENIZATION ANALYSIS METHOD

HA is here applied to the flow problem with periodic micro-structures (Sanches-Palencia 1980; Ichikawa *et al.* 1999). For this problem the Navier-Stokes equation is assumed for the local flow field.

We start with the following steady-state incompressible Navier-Stokes equations:

$$-\frac{\partial P^{\varepsilon}}{\partial x_i}+\eta\frac{\partial^2 V_i^{\varepsilon}}{\partial x_k \partial x_k}+F_i=0 \quad \text{in } \Omega_{\varepsilon f}, \tag{1}$$

$$\frac{\partial V_i^{\varepsilon}}{\partial x_i}=0 \quad \text{in } \Omega_{\varepsilon f}, \tag{2}$$

where V^{ε}_i is the velocity vector with the shearing viscosity η, P^{ε} the pressure, F_i the body force vector, and $\Omega_{\varepsilon f}$ the water flow region. The script ε implies that the functions V^{ε}_i and P^{ε} change rapidly in the small scale region because of the microscale inhomogeneity.

Let us introduce the local coordinate system $\boldsymbol{y}$ which is related to the global system, $\boldsymbol{x}$, by $\boldsymbol{y}=\boldsymbol{x}/\varepsilon$. Here, ε is a microscale geometry parameter. Since we think the limiting case $\varepsilon \to 0$, the differentiation in Eq.(1) and (2) can be changed into:

$$\frac{\partial}{\partial x_i} \Rightarrow \frac{\partial}{\partial x_i}+\frac{1}{\varepsilon}\frac{\partial}{\partial y_i} \tag{3}$$

Now asymptotic expansions for V^{ε}_i and P^{ε} are introduced as:

$$\begin{aligned} V_i^{\varepsilon}(\boldsymbol{x}) &= \varepsilon^2 V_i^0(\boldsymbol{x},\boldsymbol{y})+\varepsilon^3 V_i^1(\boldsymbol{x},\boldsymbol{y})+\ldots, \\ P^{\varepsilon}(\boldsymbol{x}) &= P^0(\boldsymbol{x},\boldsymbol{y})+\varepsilon P^1(\boldsymbol{x},\boldsymbol{y})+\ldots, \end{aligned} \tag{4}$$

where $V^{\alpha}_i(\boldsymbol{x},\boldsymbol{y})$ and $P^{\alpha}(\boldsymbol{x},\boldsymbol{y})$ (α=0,1,…) are **Y**-periodic functions such as $V^{\alpha}_i(\boldsymbol{x},\boldsymbol{y})= V^{\alpha}_i(\boldsymbol{x},\boldsymbol{y}+\boldsymbol{Y})$, $P^{\alpha}(\boldsymbol{x},\boldsymbol{y})= P^{\alpha}(\boldsymbol{x},\boldsymbol{y}+\boldsymbol{Y})$ with the size of a unit cell **Y**.

Substituting Eqs. (3) and (4) into Eq. (1) and taking $\varepsilon \to 0$ yield that the each term of ε must be zero:

ε^{-1}-term: $\qquad \dfrac{\partial P_0}{\partial y_i} = 0 \quad \Rightarrow \quad P^0(\boldsymbol{x},\boldsymbol{y}) = P^0(\boldsymbol{x}) \qquad$ in $\quad Y_f \qquad (5)$

ε^{0}-term: $\qquad -\dfrac{\partial P^1}{\partial y_i} + \eta \dfrac{\partial^2 V_i^0}{\partial y_k \partial y_k} = \dfrac{\partial P^0}{\partial x_i} - F_i \qquad$ in $\quad Y_f \qquad (6)$

where Y_f is the fluid flow region in the microscale domain.

Similarly Eq. (2) gives:

ε^{1}-term: $\qquad \dfrac{\partial V_i^0}{\partial y_i} = 0 \qquad$ in $\quad Y_f \qquad (7)$

ε^{2}-term: $\qquad \dfrac{\partial V_i^0}{\partial x_i} + \dfrac{\partial V_i^1}{\partial y_j} = 0 \qquad$ in $\quad Y_f \qquad (8)$

Since the right hand side (RHS) of Eq. (6) is a function of only the global system $\boldsymbol{x}$, we introduce a separation of variables as:

$$\begin{aligned} V_i^k &= \left(F_k(\boldsymbol{x}) - \frac{\partial P^0(\boldsymbol{x})}{\partial x_k} \right) v_i^k(\boldsymbol{y}), \\ P^1 &= \left(F_k(\boldsymbol{x}) - \frac{\partial P^0(\boldsymbol{x})}{\partial x_k} \right) p^k(\boldsymbol{y}), \end{aligned} \tag{9}$$

Here $v_i^k(\boldsymbol{y})$ and $p^k(\boldsymbol{y})$ (k = 1,2,3) are called the characteristic velocity and the characteristic pressure, respectively. Then Eq. (6) is changed into a PDE of only the local system $\boldsymbol{y}$:

$$-\frac{\partial p^k}{\partial y_i} + \eta \frac{\partial^2 v_i^k}{\partial y_j \partial y_j} + \delta_{ik} = 0 \qquad \text{in} \quad Y_f \tag{10}$$

In the similar manner the mass conservation law (7) can be written as:

$$\frac{\partial v_i^k}{\partial y_i} = 0 \qquad \text{in} \quad Y_f \tag{11}$$

Eqs. (10) and (11), called the 'microscale equations' (MiSE) for the water flow problem, can be solved under periodic boundary conditions.

Now an averaging operation is introduced for Eq. $(9)_1$ and we get the following Darcy's law in the sense of HA:

$$\tilde{V}_i^0 = K_{ji} \left(F_j - \frac{\partial P^0}{\partial x_j} \right), \qquad K_{ji} = \tilde{v}_i^j = \frac{1}{|\boldsymbol{Y}|} \int_{Y_f} v_i^j \, dy, \tag{12}$$

where $\tilde{V}_i^0$ is the averaged velocity in the unit cell ($|\boldsymbol{Y}|$, the volume of the unit cell) and K_{ji} is called the HA-permeability. It can be shown that K_{ji} is symmetrical and non-negatively definite. The same averaging is applied to Eq. (8), then the second term vanishes because of the periodic boundary condition of V^1_i, so the following 'macroscale equation' (MaSE), called the HA-flow equation, is specified:

$$\frac{\partial \tilde{V}_i^0}{\partial x_i} = 0 \qquad \text{or} \qquad \frac{\partial}{\partial x_i} \left[K_{ji} \left(F_j - \frac{\partial P^0}{\partial x_j} \right) \right] = 0 \quad \text{in } \Omega. \tag{13}$$

The true pressure P^ε and velocity V^ε_i are calculated in the first order approximation sense by Eq. (3) as

$$V_i^\varepsilon(\boldsymbol{x}) \cong \varepsilon^2 V_i^0(\boldsymbol{x},\boldsymbol{y}), \qquad P^\varepsilon(\boldsymbol{x}) \cong P^0(\boldsymbol{x}). \tag{14}$$

In geotechnical engineering we commonly use the following empirical Darcy's law:

$$\tilde{V}_i' = -K_{ij}' \frac{\partial H}{\partial x_j}; \qquad H = \frac{P}{\rho g} + \zeta, \tag{15}$$

where $\tilde{V}_i'$ is the average velocity in the classical sense (called the seepage velocity), H is the total head, P is the pore pressure, ζ is the elevation head, g is the gravitational acceleration and ρ is the mass density of water which is assumed to be constant because of its incompressibility. Comparing this with Eqs. (13)-(15), we have the correspondence:

$$\tilde{V}_i' = \tilde{V}_i^{\varepsilon} \cong \varepsilon^2 \tilde{V}_i^{0}, \tag{16}$$

so the HA-permeability K_{ij} is related with the C-permeability K'_{ij} as:

$$K'_{ij} = \varepsilon^2 \rho g \, K_{ij}. \tag{17}$$

Note that this C-permeability K'_{ij} is specified by some experimental procedures. The validity of the HA-permeability concept has been proved by several works (Ichikawa *et al*. 1999; Chae 2004a).

INPUT PARAMETERS FOR THE HOMOGENIZATION ANALYSIS

Fracture roughness

The specimens used for the HA are granites that have a single natural fracture. The fracture roughness is measured using a confocal laser scanning microscope. Sample spacing is 2.5µm in both x-and y-directions. The highest resolution in the z-direction is 0.05µm, which is more sensitive than the previous methods (Chae *et al.* 2004b).

The 3-D configuration of roughness as well as the 1-D roughness profile is measured for each specimen. The resolutions in the x- and y-directions are fixed as 1,024 x 768 pixels (2.56 x 1.92 mm in area) and the resolution of z-direction is 10 µm.

The Fourier spectral analysis is conducted to quantitatively identify roughness characteristics (Chae *et al.* 2004b). After the spectral analysis and noise filtering is completed for all of the data for each specimen, a reconstruction of the roughness geometry is performed using only the influential frequencies among the components (Fig. 1). The reconstructed roughness profiles are used for the fracture models in the HA numerical simulation.

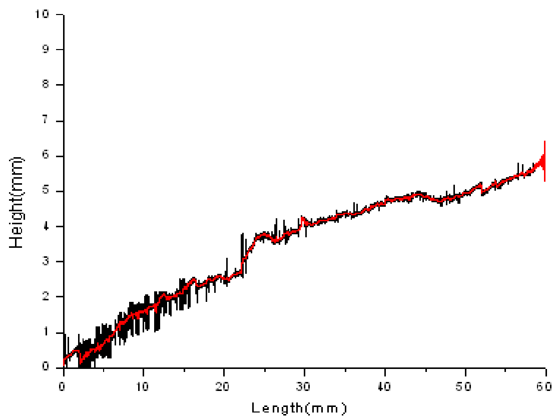

Figure 1. An example of roughness patterns that show both noises (black) and the smoothed roughness data (red).

Aperture variation dependent upon the uniaxial compression

The fracture apertures are also used as input parameters for the HA simulation. They are measured by the CLSM while applying normal stress (Chae *et al.* 2003). Among all of the aperture data, three stress levels (10, 15 and 20 MPa) are applied. The mechanical apertures are equal to the mean value of the measured apertures for each specimen using the CLSM. The hydraulic apertures are calculated using an equation satisfying the cubic law (Zimmerman and Bodvarsson 1996). The hydraulic conductivities are both calculated with the mechanical aperture, the measured aperture by the CLSM, and the hydraulic aperture are also calculated from an equation based on the cubic law (Eq. 7).

$$K_f = k_f \frac{\gamma}{\mu} = \frac{e_h^2}{12} \frac{\gamma}{\mu} \tag{7}$$

where k is intrinsic permeability coefficient, γ is unit weight of water, μ is viscosity of water, e_h is hydraulic aperture, and L is length of specimen.

COMPUTATION OF PERMEABILITY USING THE HA UNDER VARIOUS FRACTURE CONDITIONS

The 2-D fracture models are now constructed for the HA simulation. The computation is performed assuming a temperature condition of 300K. The water viscosity, η is equal to 0.8×10^{-3} Pa · sec and the mass density, ρ is equal to 0.99651g cm^{-3}. The HA permeability characteristics are shown under various roughness and aperture conditions. That is, under various types of observed roughness features the upper fracture wall is displaced at intervals of every 1 mm in the shearing direction. This shear displacement is introduced for five stages, which results in various aperture values along the fracture. Permeability is calculated at every stage of the displacement.

An example of the fracture models are shown in Fig. 2. These models represent various roughness features and aperture due to the displacement. Every model shows different geometrical features at each stage of the displacement. The calculation results and the relationships between the square of the mean aperture, b^2, and the calculated permeability are shown in Fig. 3. It is found that the permeability coefficients are irregularly ranged from 10^{-4} to 10^{-1} cm/sec, while the coefficients of the previous parallel plate models are uniformly distributed in some range. This is due to the complicated change of the aperture as increasing the shear displacement in the current models. In this figure it is not possible to find any relationship, so the cubic law is not suitable for the rough fracture case (Chae *et al.* 2004a).

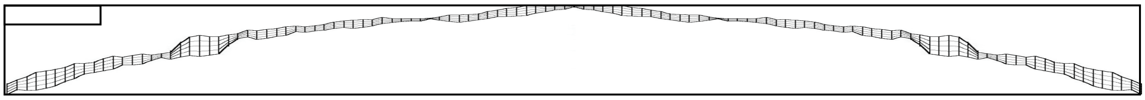

Figure 2. An example of fracture models showing various roughness and apertures on each stage. Exaggerated 50 times in vertical direction.

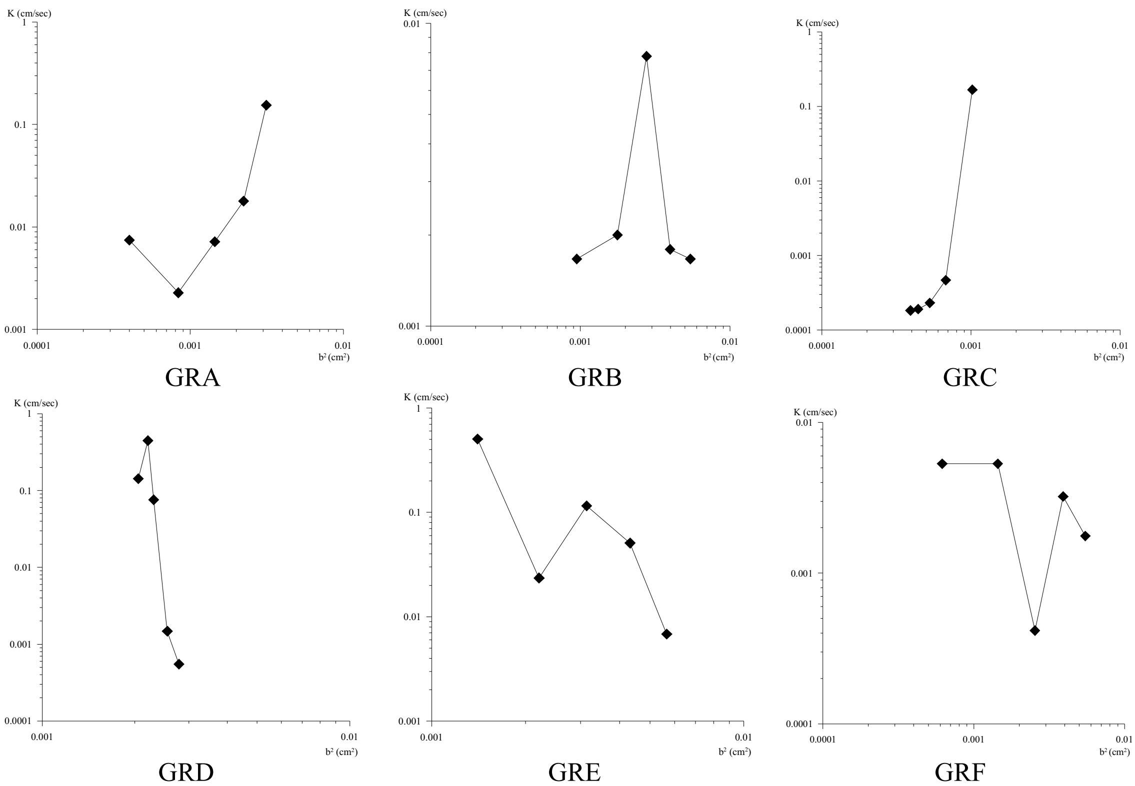

Figure 3. Relationship between permeability coefficients and aperture square.

CONCLUSION

Considering the change of aperture and roughness pattern simultaneously along a fracture, the permeability is calculated by using the rough fracture models. The upper wall is assumed to be displaced by shearing in the five stages. The calculation results show various changes of permeability which depend on the roughness patterns and aperture values. It is understood that the cubic law is not appropriate for the fracture with rough walls. The irregular distribution of aperture along a fracture may introduce a negative proportional relationship between the aperture and the permeability even though the mean aperture becomes larger. This clearly proves that fracture permeability is very sensitive to the geometry of the roughness and the aperture of the fracture. The approach will be effectively applied to the analysis of permeability characteristics as well as the fracture geometry in discontinuous fracture rock masses.

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